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Three Essays on the Economic Theory of Mating and Parental Choice

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Declaration

The candidate hereby declares that

- the thesis has been composed by the candidate, and
- that the work is the candidate's own, and
- that the work has not been submitted for any other degree or professional qualification except as specified.

signed

Abstract

Chapter 1: Relative Concerns and the Choice of Fertility

Empirical research has shown that people exhibit relative concerns, they value social status. If they value their children's status as well, what effect will that have on their decisions as parents?

This paper argues that parents and potential parents are in competition for status and rank in the generation of their children; as a consequence richer agents may cut back on the number of children they have and invest more in each child to prevent children of lower income agents from mimicking their own children. This effect need not be uniform so that equilibrium fertility may e.g. be a U-shaped function of income, even when agents would privately like to increase fertility when they receive greater income.

These findings have wide ramifications: they may contribute to our understanding of the working of the demographic transition; they also suggest that the low fertility traps seen in some developed countries are rather strongly entrenched phenomena; and they offer a new explanation for voluntary childlessness.

Chapter 2: Relative Concerns and Primogeniture

While pervasive in the past, differential treatment of children, i.e. different levels of attention and parental investments into children of the same parent, has become rare in modern societies. This paper offers an explanation based on technological change which has rendered the success of a child more uncertain for a parent who is deciding on how much to invest into each of his children.

Within a framework of concerns for social status (or relative concerns), agents decide on how many children to have and how much to invest in each child. When their altruism towards each child is decreasing in the total number of children, it is shown that they may solve the trade-off between low investment, high marginal return children (that come in large numbers and hence hurt parental altruism) and high investment, low marginal return children (that come in low numbers) by demanding both types and hence practice differential treatment. Uncertainty over status or rank outcomes of children reduces the range of equi-

librium investment levels into children so that the difference in the numbers they come in is reduced. Eventually the concern for return dominates and differential treatment disappears.

Chapter 3: Co-Evolution of Institutions and Preferences: the case of the (human) mating market

This paper explores the institutions that may emerge in response to mating preferences being constrained in their complexity in that they can only be conditioned on gender not other characteristics of the carrier of the preferences. When the cognitive capacity of the species allows a sophisticated institutional setup of one gender proposing and the other accepting or rejecting to be adopted, this setup is shown to be able to structure the mating allocation process such that preferences evolve to forms that, conditional on the setup, are optimal despite the constraint on complexity. Nature can be thought of as delegating information processing to the institutional setup.

In an application to humans it is shown that the mechanism of the model can help explain why men and women may exhibit opposed preferences in traits such as looks and cleverness. The anecdotal fact that women do not marry down while men do can be interpreted as a maladaptation of female preferences to modern marriage markets.

Chapter 1

Relative Concerns and the Choice of Fertility

In all civilised countries man accumulates property and bequeaths it to his children. So that the children in the same country do not by any means start fair in the race for success.

Charles Darwin [1871], Vol.1 p. 169.

1.1 Introduction

In an online contribution to a well-established British newspaper, a couple asks other readers to advise them on whether or not to have additional children:

My husband and I have two wonderful children and are thinking of having four or even five, as we're both from large families ourselves. But we're not that well-off and our friends say we're mad. *Kids today are very demanding and you can't expect them to go without the things other children have.* Is it really an impossible (financial) dream?¹

This contribution is interesting for a number of reasons: firstly, the couple consciously decides on whether or not to have an additional child and seems to

¹See guardian.co.uk [2006], italics added.

do so by weighing benefits against costs, which implies that fertility can be seen as the result of economic decision-making.

Secondly, the sentence in italics appears to indicate, at least in my view, that in their calculation about how to endow the potential additional children the couple references a child's needs by the endowment their peer couples give to their children. It seems plausible that the high endowments they witness other couples giving to their (few) children makes this couple think about the large family they would like to have as not affordable, as “an impossible (financial) dream”.

I venture to hypothesise that the couple provides an example of the following logic: people care about the well-being of their offspring and they anticipate that this well-being depends partly on the relative success of their children in life; as a consequence they try to favourably influence their children's success by investments, endowments or bequests of various forms. The choice of family size and parental investments in children is thus necessarily part of a strategic game among potential parents².

The present paper explores this hypothesis and its effects on fertility behaviour.

In the following, a model of endogenous fertility choice is developed. Preferences are chosen such that in the absence of endogenous status the trade-off between number of children and parental investment per child, or quantity and quality of children, is balanced in the sense that an increase in income leads to a higher demand for both.

Equilibria with endogenous status are then characterised³. The possibility of influencing the status of one's children via manipulating the parental investment has strong implications for the fertility behaviour of agents.

In equilibrium, the number of children is never higher than the privately optimal number, and it is a decreasing function of how intense status competition is. Low

²The idea of this chapter and its literature review build on work the candidate undertook for his MSc dissertation of 2008.

³As to the motivation compare Frank [1985b]: “[...] an element of almost overriding importance in the structure of human motivation will be a taste for seeing to it that one's children are launched in life as successfully as possible. Now, how successful one's children will be in life depends much less on their skills and endowments in any absolute sense than on how these compare with the skills and endowments of others” (p. 102).

income differences and large status differentials between income groups make for fiercer competition: agents of lower status are more able and better motivated to mimic the rich in the way they endow their children. Agents of higher status and the rich respond by cutting back on the number of children they have and by investing inefficiently high amounts in their ‘remaining’ children. Conditions are derived under which competition is intense enough to bring about these effects on fertility.

It is then shown that the above logic is capable of producing non-monotonic patterns in the fertility and income relationship. The middle class may feel a stronger mimicking threat from the poor than do the rich from the middle class; the result can be a U-shaped fertility pattern in which lower and upper ends of the income distribution have a greater number of children than the middle class, which may fall back to having single children.

Furthermore, it can be shown that under certain conditions agents from the middle of the distribution may choose to opt out of the game completely by not reproducing at all.

The analysis has various implications. It contributes to our understanding of fertility behaviour in societies that allow for social mobility; and it adds the channel of greater economic equality by which economic development may bring about a demographic transition. Further, it may add to the explanation of the abundance of single child families among the middle class in many developed countries and of the consistently reported fall of fertility in focal regions of economic development in the less rich countries, i.e. cities.

In the normative area, the externality that agents of lower status impose on agents of higher status by pushing parental investment to inefficiently high levels may provide a case for government intervention. It is intriguing to think that, with the intention to exogenise child status, societies may find it opportune to socialise investments and expenditures on children by which their later income is influenced, even though the individual intergenerational care that this would try to crowd out has possibly been the main thrust behind most real and human capital formation.

The paper is organised as follows: the next section reviews the relevant literature on relative concerns (or desire for status) to which this paper contributes;

then the model setup is presented and its implications examined. The final section concludes.

1.2 Related Literature

Neoclassical economic analysis presupposes that the economic agent derives utility from the bundle of goods he consumes irrespective of what other agents consume. This view of man is analytically convenient and provides a very pure as well as concise and thus defensible conception of human motivation.

Many economists, however, have argued that modelling preferences according to this view does not capture an important apparent feature of economic behaviour: people seem to have what has become known as ‘relative concerns’, i.e. people care not only about how much they consume in absolute terms but also about how their consumption, income and other characteristics compare to those of others.

The idea traces at least as far back as Thorstein Veblen’s seminal book “The Theory of the Leisure Class” of 1899 in which he argued that in a tradition that reaches back to the stone ages people seek status, i.e. rank in social hierarchy, and that they are willing to part with money in attempts to favourably change other people’s perception of themselves. These attempts mean that people try to copy the behaviour of higher-status people who display their status by engaging in what Veblen called “conspicuous leisure” and “conspicuous consumption”⁴, i.e. signalling-type wasteful activities.

The first formal model of relative concerns can be found in Duesenberry [1949]; his model can be interpreted as one in which people care about their total consumption and how it compares to average consumption of the population. This is sometimes called the comparison with “the Joneses”.

If the utility derived from the consumption of a good was equally affected by relative concerns for all goods, then the behaviour of people would not be affected⁵; if, however, the effects were unequal we may evidence a different behaviour from the one that would prevail in the absence of relative concerns. In

⁴Compare Veblen [1899], chapter 3 and 4, respectively.

⁵Compare Solnick and Hemenway [2005], p. 147.

this regard, the literature refers to Hirsch [1978]⁶ who distinguished positional from non-positional goods. The former are defined in the following way: “The positional economy [...] relates to all aspects of goods, services, work positions, and other social relationships that are either (1) scarce in some absolute or socially imposed sense or (2) subject to congestion or crowding through more extensive use.”⁷

The seminal article to model relative concerns in an economy with positional and non-positional goods is Frank [1985b] who defines the rank in the consumption of the positional good as the variable of interest regarding relative concerns. He shows that people consume too much of the positional good (and accordingly too little of the non-positional good) relative to the pareto-optimal solution. This result has been replicated in many different guises in the ensuing literature⁸ (including this paper).

The literature divides into three ways of motivating relative concerns. On one side are those who think of relative concerns as a predisposition that was moulded by natural selection during the evolution of man; this includes for instance Frank [1985b]⁹.

Samuelson [2004] provides a rationale for the claim that relative concerns are an intrinsic feature of human beings; he shows that in an evolutionary setting nature may select agents that (imperfectly) infer the state of the environment from other agents’ consumption. If fitness depends on the state of the environment it may follow that in the evolutionary optimum agents maximise their relative standing instead of the absolute level of consumption.

This line of thought was taken further e.g. by Rayo and Becker [2007] who embed the idea that the consumption of other agents provides a signal about the state of nature¹⁰ into the setting of a fitness-maximising design of the utility function.

On the other side are those who argue that these concerns should be in-

⁶Compare for instance Frank [1985b] p. 101 and Hopkins and Kornienko [2004] p. 1088.

⁷See Hirsch [1978], p. 27.

⁸Compare for instance Ireland [1994], Hopkins and Kornienko [2004] and Frank [2005]

⁹Compare also Frank [1985a]: “Evolutionary forces saw to it that people come into the world with a drive mechanism that makes them seek to outrank others with whom they compete for important resources” (p. 268).

¹⁰Rayo and Becker [2007] speak of “common productivity shocks” (p. 324).

terpreted as instrumental in that people care for how they compare with others not for the comparison's sake but because occupying a favourable social position entails access or better access to goods the agent cares for.

The idea is that relative comparisons replace market mechanisms where there are market imperfections, missing markets or non-pecuniary aspects of transactions. Status is sought for the same reason for which pecuniary wealth is sought: it can be used to acquire valued goods. This understanding is elaborated in Postlewaite [1998]; and an example of the instrumental interpretation is provided in Cole et al. [1992] who consider a marriage market that matches men and women who differ in wealth and gender-specific endowment, respectively; they show that status enters the utility function in the reduced form although it has no intrinsic value to the agents¹¹.

The third party occupies the middle ground by remaining agnostic about the source of these relative preferences; the modelling of relative concerns is motivated by the fact that they can be observed empirically. This party includes e.g. Hopkins and Kornienko [2004] and the present paper.

The empirical evidence comprises papers such as J. Solnick and Hemenway [1998], Solnick and Hemenway [2005] who have conducted surveys in order to identify what goods or characteristics people regard as rather positional and which as rather non-positional. Their findings strongly suggest that people exhibit relative concerns and do so to a different degree in different aspects of life¹².

The implications of relative concerns are both wide in scope and deep in consequence. Recent studies that explore these implications include Gali [1994] who studies asset markets effects. He takes the comparison of own consumption to average consumption as the object of relative concern as Duesenberry [1949] and finds that average consumption influences the risk taking behaviour of agents through its impact on marginal utility of own consumption.

Neumark and Postlewaite [1998] find empirical support for their hypothesis that

¹¹Compare Cole et al. [1992], pp. 1114-19.

¹²J. Solnick and Hemenway [1998] go further and suggest that one “might be able to rank people by concern for relative standing in the same way that they are classified by their time preference or level of risk aversion. Attitude toward risk is considered an important dimension along which people differ. Attitude toward relative standing may be equally important in affecting satisfaction and behaviour.” (Compare p. 380).

the decision to take a job is influenced by the comparison of the income of a married woman's household to that of her sister-in-law's contributing to the explanation of the rise in the proportion of married women who choose to work in the market sector.

Bagwell and Bernheim [1996] take a signalling equilibrium where relative concerns are assumed to be motivated by preferential treatment by social contacts and show that under special circumstances these concerns give rise to "Veblen effects" which "are said to exist when consumers exhibit a willingness to pay a higher price for a functionally equivalent good"¹³.

Drawing on findings of auction theory¹⁴, Hopkins and Kornienko [2004] investigate the strategic interactions resulting from relative concerns in the form of concern for the rank in the consumption of a positional good and derive comparative statics results when the income distribution changes¹⁵.

In a multi-generational context, the implications of relative concerns have been studied by Moav and Neeman [2010] who show that when both conspicuous consumption and level of human capital signal wealth and thus confer status for which agents care intrinsically, then the budget share of conspicuous consumption decreases with the level of human capital; this may give rise to the existence of a poverty trap for families that start at low ranking incomes.

While Moav and Neeman [2010] fix fertility exogenously and study the implications of relative concerns on economic development given this fertility, the present paper takes the income process as exogenous and studies the implications of relative concerns on endogenous fertility behaviour. Status of children is taken to depend positively on parental investment while being unrelated to the number of children an agent has; hence investment in children is positional while fertility is non-positional and thus potentially moves to levels below the socially and privately optimal in response to competition for status.

The present paper naturally also builds on the economic theory of fertility.

¹³Compare Bagwell and Bernheim [1996], p. 349.

¹⁴The resemblance of the allocation mechanisms of strongly positional goods to auctions was already discussed in Hirsch [1978], pp. 28-29.

¹⁵Similar research questions were investigated in Ireland [1994]; Ireland, however, conducts a large part of his analysis for the special case of quasi-linear preferences. Moreover, consistent with the signalling idea of his model, status is defined over absolute wealth whereas in Hopkins and Kornienko [2004] only the relative standing matters.

In particular it can be seen as following the tradition of what might be called the ‘Chicago School’ in fertility theory; this tradition takes preferences as exogenous and stable, postulates a household production function which transforms market goods and time into final goods like children and their attributes. Starting with Becker [1960], the school emphasises the distinction of and trade-off between quantity and quality of children; see Becker and Lewis [1973] and Becker and Tomes [1976]. The model of this paper extends this trade-off by adding the channel of status to the quality side.

The building blocks of the approach of this paper can be found in the literature: Easterlin [1976] hypothesises that parents try to secure a status at least as high as theirs for their children¹⁶; an exposition on intergenerational social mobility can be found e.g. in Becker [1981]; Sah [1991] restricts fertility choice to integer numbers; and strategic reasoning has been explored in the context of bequests by Bernheim et al. [1985], although to a different purpose¹⁷. Closest to the present paper is Tournemaine [2008] who introduces both relative concerns and endogenous fertility and bequests as a feature into an “R&D-based model”. His different research interest, however, prevents a more than cursory examination of relative concerns; the main obstacle being that his model assumes homogeneous agents so that relative concerns are somewhat artificial. The specific research question and technique is new to the best knowledge of the author.

1.3 Model with Exogenous Fertility

The focus of the paper is on how the urge for status shapes the economic decisions of agents in the intergenerational domain; attention is therefore restricted to these decisions. First, we may thus keep the model simple by taking the

¹⁶Compare Easterlin [1976]: “There is considerable evidence that an important concern of nineteenth century American farmers was to give their children a ‘proper’ start in life, a start that would enable the children to enjoy over their lifetime a socioeconomic status comparable to that in which they were raised” (p. 422).

¹⁷Bernheim et al. [1985] examine how individuals may try to influence the behaviour of children and other close relations by making bequests contingent on the behaviour of these. In contrast, I study how parents choose to endow their children when knowing that they are strategically interacting with other parents.

income of agents to be completely determined by how much their parents invested in them (via, among other things, monetary bequests, provision of higher education and time dedicated to caring and teaching); this renders own income an exogenous variable for the agents. They are thus solely concerned about how to spend it and are assumed to use it to pay for their own consumption of a composite good and for investment in their own children.

As for the intergenerational framework, I build on Becker and Barro [1988] and so assume that each agent maximises a dynastic utility function into which the consumption of the agent and that of all his descendants enters in an additively separable manner¹⁸. One may imagine the agent, who could be interpreted as a couple or household, to value the wellbeing of his descendants for intrinsic reasons like family based altruism, which in turn may for instance be motivated by evolutionary arguments. For simplicity, the agent is modelled to consume and asexually reproduce at a single point in time.

Imagine a world of two generations of agents and imagine that each parent agent has exactly one child. Parent agent utility can be expressed as:

$$V_i(z_i) = \max_{b_i} \left[u(z_i - b_i) + \alpha u(qb_i) \right] \quad (1.1)$$

Where z_i stands for the parent agent's income, b_i is the bequest or parental investment into the single child, u is a standard, strictly increasing and concave consumption utility function that is twice differentiable and satisfies $u(0) = 0$, $\alpha \in (0, 1)$ is an altruism parameter and $q > 0$ is a productivity parameter of parental investment. The decision problem of the parent agent is concave so that there is a unique $b_i \in (0, z_i)$ that maximises V_i . Note that the optimal parental investment b_i is increasing in parental income z_i .

Children simply consume their income and receive a utility of $u(qb_i)$.

The population of parent agents under study is assumed to be of measure 1 and to be divided into a finite number of income groups. Each income group has a positive measure of agents and each income group is associated with a different level of income. This associated level of income is the common income level of all the agents in the income group. The income distribution is thus assumed

¹⁸I depart from Becker and Barro [1988] by focusing on the two generation case and by restricting fertility choice to integer numbers; which makes the equilibrium predictions both easier to derive and sharper.

to be discrete; the support being the set of income levels associated with the income groups. It should be noted that an individual agent has measure 0.

I denote the set of income levels associated with the income groups as Z_p ; I shall refer to Z_p as the set of parent incomes because it contains all the income levels that can be observed among parent agents. Let the elements be ordered in decreasing manner such that $z_i > z_{i+1}$ where z_i and z_{i+1} are elements of Z_p .

Now I introduce ‘relative’ or ‘status’ concerns. Depending on their position in the income distribution, agents enjoy a certain ‘status’ which is a scalar number taken from a set of stati denoted \hat{S} ; let the elements of \hat{S} be ordered by their value such that $s_1 > s_2$ and so on. In the parent generation, status is awarded to income levels by the status function $S_p : z \rightarrow r$; in the child generation the status function is called S_c . In either generation, this status function is a step function and the levels of income at which its value jumps shall be called income thresholds. Let the set of income thresholds in the parent generation be denoted by \check{Z}_p and in the child generation by \check{Z}_c , an element being denoted by $\check{z}_{p,i}$ and $\check{z}_{c,i}$, respectively.

Agents whose income is equal to or surpasses the highest status threshold in their generation are assigned the highest status s_1 ; agents whose income is greater than or equal to the next highest but lower than the highest threshold in their generation are assigned the second highest status s_2 . In general, in generation $x \in \{p, c\}$ we can find for any income level z the next lower status threshold, say $\check{z}_{x,s}$ with $\check{z}_{x,s-1} > z \geq \check{z}_{x,s}$. Then S_x assigns this income level z the status associated with this next lower threshold, i.e. s_s .

Next, I let the status thresholds be defined in each generation by the income levels of the income groups in that generation, so e.g. we have $\check{Z}_p = Z_p$ in the parent generation. S_c is thus only defined if the child income distribution is discrete.

The number of income groups and stati may not match. If there are more stati than income groups I assume that the lowest stati are not assigned while if there are more income groups then the income groups with the lowest income levels share the lowest status. We can formalise this for the example of the parent generation: for $|Z_p| \leq |R|$ we have $S(z_i) = s_i \forall z_i \in Z_p$, while for $|Z_p| > |R|$ we have $S(z_i) = s_i$ for $i \leq |R|$ and $S(z_i) = s_{|R|}$ for $i > |R|$. This way of assigning stati is illustrated in Figure 1.1.

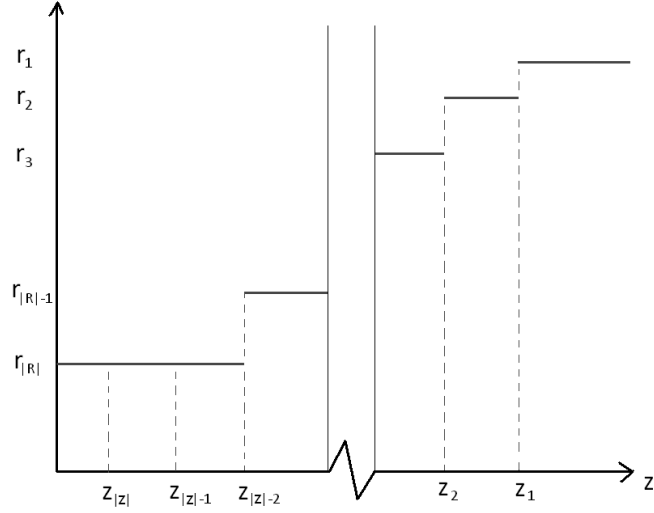


Figure 1.1: *Graphical representation of the status function in the parent generation, S_p , given the set of parent income, Z_p , and the set of stati \hat{S} in the case of $|Z_p| > |R|$. Note that status depends neither on the distance between these income levels nor on how other agents are distributed amongst stati.*

In what follows, I write $s_{p,i} \in \hat{S}$ for the equilibrium status that the parent agent of income z_i occupies and $s_{c,i} \in \hat{S}$ for the equilibrium status of his child or children.

Following a formulation pioneered by Frank [1985b] and recently advanced by Hopkins and Kornienko [2004], direct consumption utility and status enter the utility function in a multiplicative manner. Let the instantaneous utility function thus be given not by u but by $u \cdot S$ where S stands for status as defined above.

We can update the utility function of a parent agent as follows:

$$V_i(z_i) = \max_{b_i} \left[u(z_i - b_i) s_{p,i} + \alpha u(qb_i) S_c(qb_i) \right] \quad (1.2)$$

Note that, when considering deviating to out-of-equilibrium levels of parental investment, the parent agent takes S_c as given as, being of measure 0, his decision does not influence the income distribution in the child generation.

I now illustrate the implications of status concerns as defined in this setup with two simple examples.

Example 1 A WORLD WITHOUT INCOME DIFFERENCES

Suppose there is only one income group in the parent generation. Suppose further that parent agents coordinate on a symmetric pure strategy Nash equilibrium. It is then clear that whatever level of parental investment they coordinate on, there is only one observed income level in the child generation so that all children enjoy the highest status (as do their parents).

The interesting bit to notice here is that the level of parental investment may be higher than the Pareto optimal one which is given by the solution to the first order conditions of Equation (1.2): $-u'(z_i - b_i)s_1 + \alpha u'(qb_i)qs_1 = 0$. Suppose the agents coordinate on a level of parental investment slightly higher than the Pareto optimal one; then any lower investment level (including the Pareto optimal one) would let the child receive status s_2 instead of s_1 . The collective parental investment decisions of the parent agents can thus impose negative external effects on parent agents even though the decision of any individual agent imposes no externalities.

Of course the equilibrium parental investment level cannot be lower than the Pareto optimal one as a deviation to the latter would not result in a status loss and would therefore be profitable.

Example 2 A WORLD WITH TWO PARENT INCOME GROUPS

Suppose now that there are two income groups in the parent generation. Suppose again that parent agents coordinate on a symmetric pure strategy Nash equilibrium. Finally, suppose that the income difference between the two groups is so large that the Pareto optimal parental investment level of the rich parent agents is at least as high as the income of the poor parent agents. This way, irrespective of what levels of parental investment levels the agents coordinate on, a poor agent cannot hope to emulate the parental investment decision of a rich agent and there is thus no link between the decisions of agents of one and the other income group.

Now suppose that the income difference between the parent agent income groups is small. Suppose further that the status difference $s_1 - s_2$ is large enough so that a poor parent agent prefers investing the level of parental investment that is Pareto optimal for rich agents to the level of parental investment that is Pareto optimal for poor agents given that their children receive status s_2 if the former level lets his child receive a status of s_1 instead of s_2 . Then several types of

symmetric pure strategy Nash equilibria are possible:

- *Parent agents may coordinate on the parental investment level that is Pareto optimal for rich agents so that children of all agents have the same income level and status.*
- *Alternatively, poor agents may coordinate on a parental investment level that is below the one that rich agents coordinate on which in turn is higher than the one that is Pareto optimal for rich agents. This forms an equilibrium if the parental investment level chosen by rich agents is high enough to make a status s_1 child excessively expensive for poor agents.*

Note that if the income difference between parent agent income groups is small then the latter type of equilibrium may not exist.

These two examples have shown that the urge for competition can lead to inefficient equilibrium outcomes: parents overinvest into their children and they do so out of fear that their children may fall behind the children of their peers. In the following I introduce endogenous fertility choice and show that the burden of this inefficiently high parental investment may partly fall onto fertility leading to inefficiently few children.

1.4 Model with Endogenous Fertility

We now turn to a version of the model that allows for endogenous fertility which I restrict to integer numbers. The utility function of a parent agent of income group i can now be written as:

$$V_i(z_i) = \max_{b_i, n_i} \left[u(z_i - n_i(b_i + \beta))s_{p,i} + \alpha \cdot n_i^{1-\epsilon} \cdot u(f(b_i; z_i))S_c(f(b_i; z_i)) \right] \quad (1.3)$$

The dynastic or overall utility function of the parent agent is thus the sum of his instantaneous utility and that of his n_i child(ren); the instantaneous utility of his child(ren) enters weighted by $\alpha \cdot n_i^{-\epsilon}$ with $\alpha \in (0, 1)$ and $\epsilon \in [0, 1)$. This weighting reflects the idea that, other things equal, the agent values his own

consumption more than that of his children and exhibits a decreasing degree of altruism towards each individual child the more children he has; one may think of α as the altruism parameter that measures how much one generation values the next and of ϵ as a dilution parameter that measures how strongly the number of siblings diminishes the parental affection towards the individual child¹⁹.

Generalising qb_i of the preceding section I write $f(b_i; z_i)$ for the twice differentiable and strictly increasing function mapping parental investment into a child, b_i , to the income of that child where $f(0; z_i) = 0$; f is parameterised by the income of the parent agent, z_i . I further write $\beta > 0$ for some fixed cost of having a child. I assume that parent agents treat their children equally, i.e. invest the same amount in each; consequently, all children of an agent enjoy the same status. For an analysis that permits differential treatment of children see Chapter 2.

I introduce a fixed cost of rearing a child, $\beta > 0$. This cost can be thought of as very small and is included only to ensure an interior solution in the exogenous status case.

The agent trades off three goods: the number of children he has (n_i), his parental investment (b_i which I interpret as being a per child figure), and his own consumption ($z_i - n_i(b_i + \beta)$).

In the equilibrium analysis it is going to be important that, while $s_{p,i}$ is exogenous to the agent, he may change the status his child is going to occupy by varying parental investment b_i . If the agent chooses to invest less than do his peers, i.e. other members of his income group, then his child will suffer a loss in status; if on the other hand, the agent chooses to increase b_i to the level on which agents of a higher income group coordinate, then his child will enjoy the status of those ‘richer kids’.

Parent agents are thus playing the following game. They simultaneously choose the number of children to have, n_i , and how much to invest into an individual child, b_i . The resulting distribution of child income levels determines the income thresholds of the child generation and thus the status of the children. In equilibrium this process must be in line with the expectations of the parent

¹⁹Readers who are unconvinced by the notion of dilution should note that the ensuing analysis encompasses the case of $\epsilon = 0$.

agents.

I shall focus on symmetric pure strategy Nash equilibria in which parent agents of different income groups choose different levels of parental investment so that income differences are passed on from one generation to the next. The motivation for this focus is that this setting induces the forces of competition for status that this paper seeks to explore.

1.5 Privately Optimal Choice

Before moving on to the equilibrium analysis it is useful to examine the privately optimal decision of the agent, i.e. his decision in the case that children's status is exogenous. This is done in this section.

Relaxing for a moment the integer constraint on n_i and presuming an interior solution we can take the first order conditions:

$$-(b_i + \beta) \cdot u'(z_i - n_i(b_i + \beta))s_{p,i} + \alpha(1 - \epsilon)n_i^{-\epsilon}u(f(b_i))s_{c,i} = 0 \quad (1.4)$$

$$-n_i \cdot u'(z_i - n_i(b_i + \beta))s_{p,i} + \alpha n_i^{1-\epsilon}u'(f(b_i))f'(b_i)s_{c,i} = 0 \quad (1.5)$$

The second order conditions are satisfied for $u(\cdot)$ sufficiently concave; see Appendix. Letting $\sigma_{k,l}$ denote the elasticity of k to l , and v_p and v_c denote the first and second summand of $V_i(z_i)$, respectively, we can rearrange the first order conditions in elasticities to yield the following expression:

$$\frac{b_i}{b_i + \beta}(1 - \epsilon) = u'(f(b_i))f'(b_i)u(f(b_i))^{-1}b_i \quad (1.6)$$

$$\text{and so } b_i = \frac{\frac{\sigma_{v_c,b}}{1-\epsilon}}{1 - \frac{\sigma_{v_c,b}}{1-\epsilon}}\beta \quad (1.7)$$

This, in essence, is the neoclassical result of $\frac{(\frac{\partial U}{\partial x_i})}{(\frac{\partial U}{\partial x_j})} = \frac{p_i}{p_j}$ in elasticities; to see this write E_c for $n_i(b_i + \beta)$ and E for the total expenditure of the parent agent, and note that $\frac{\sigma_{V_i,b_i}}{\sigma_{V_i,n_i}} = \frac{\sigma_{V_i,v_c}\sigma_{v_c,b_i}}{\sigma_{V_i,v_c}\sigma_{v_c,n_i}} = \frac{\sigma_{v_c,b_i}}{1-\epsilon} = \frac{b_i}{b_i + \beta} = \frac{\sigma_{E,E_c}\sigma_{E_c,b_i}}{\sigma_{E,E_c}\sigma_{E_c,n_i}} = \frac{\sigma_{E,b_i}}{\sigma_{E,n_i}}$ can be rearranged to yield the above formula for b_i .

For this interior solution to exist we need to impose an upper bound on the

product of $\sigma_{v_c, f}$ and σ_{f, b_i} so that the following inequality holds²⁰:

$$\widehat{\sigma_{v_c, b_i}} = \widehat{\sigma_{v_c, f} \cdot \sigma_{f, b_i}} < 1 - \epsilon \quad (1.8)$$

From the model setup follows that $\sigma_{v_c, f}$ is independent of z_i . f , however, is parameterised by z_i , so it may be that σ_{f, b_i} is increasing in z_i - i.e. parental investment is more efficient the richer the parent agent, which is plausible if high income is correlated with better experience in human capital building, with better contacts to potential mentors etc. - in which case the optimal b_i is increasing in the income of the parent.

Using the first order condition (1.4) and applying the implicit function theorem we can show that n_i would then be likewise increasing in the income of the parent unless b_i grows too fast with z_i in a sense formalised below; for the proof refer to the Appendix.

$$\frac{\partial b_i}{\partial z_i} < -\frac{\partial^2 V}{\partial n_i \partial z_i} \bigg/ \frac{\partial^2 V}{\partial n_i \partial b_i} \Rightarrow \frac{\partial n_i}{\partial z_i} = -\frac{\frac{\partial^2 V}{\partial n_i \partial z_i} + \frac{\partial^2 V}{\partial n_i \partial b_i} \cdot \frac{\partial b_i}{\partial z_i}}{\frac{\partial^2 V}{\partial n_i^2}} > 0 \quad (1.9)$$

I set $f(\cdot)$ such that σ_{f, b_i} is slowly increasing in z_i as discussed in the Appendix so that agents would like to invest more in each of their children and have more children when they become richer. The marginal utility gain both from higher b_i and from higher n_i is diminishing in the levels of the variables, and the marginal utility of parent consumption is also decreasing, it follows that parent consumption is also increasing in parent income. In what follows I assume that f has these properties.

Reinstating the constraint that n_i can only take non-negative integer values, it is easy to see that the implication of rising income leading to increased desire for own consumption, investment per child and number of children is preserved; the difference being that fertility changes are now lumpy and investment in children is in a sense more volatile since it takes on more of the burden of equalising the marginal contribution to overall utility across generations.

²⁰Cf. Becker and Barro [1988], equation (11) on p. 7.

Instead of being defined by (1.7), the privately optimal, or efficient, b_i can now be determined in two steps. These make use of the following two lemmas.

Lemma 1 EFFICIENT PARENTAL INVESTMENT b_i^e

Define $b_i^e(n, s_c)$ as the optimal choice of b_i given the values of n and s_c .

Then $b_i^e(n, s_c)$ can be obtained by solving

$$-n_i \cdot u'(z_i - n_i(b_i + \beta))s_{p,i} + \alpha n_i^{1-\epsilon} u'(f(b_i))f'(b_i)s_{c,i} = 0$$

which is the first order condition (1.5).

$b_i^e(n, s_c)$ can be shown to be decreasing in n ; it holds further that:

$$\begin{aligned} \frac{\partial b_i^e(n, s_c)}{\partial z_i} &> 0 \\ \frac{\partial b_i^e(n, s_c)}{\partial s_c} &> 0 \end{aligned}$$

Proof: See Appendix. □

Lemma 2 EFFICIENT NUMBER OF CHILDREN n_i^e

Define $n_i^e(b, s_c)$ as the optimal choice of n_i for fixed values of b and s_c .

It is given by the largest integer for which the net marginal benefit of an additional child is weakly positive. It can be shown that $n_i^e(b, s_c)$ is increasing in s_c and z_i , and it is decreasing in b for $b > b_i^e(n, s_c)$.

Proof: See Appendix. □

The privately optimal choice of the agent now can be characterised by the choice of n_i such that $n_i = n_i^e(b_i^e(n_i, s_c), s_c)$ and by choosing $b_i = b_i^e(n_i^e, s_c)$; let this joint optimal choice of b_i and n_i be denoted by $b_i^e(s_c)$ and $n_i^e(s_c)$, respectively.

Albeit in a ‘spiky’ manner, the agent thus invests more in each of his children and has more children as his income grows. ‘spikiness’ means that b_i^e fluctuates around the value it would take if fertility was continuous.

This can be demonstrated formally: notice that if under the continuous n regime

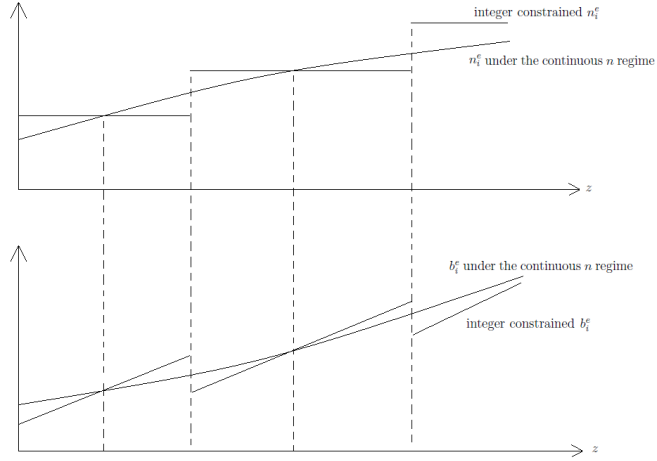


Figure 1.2: *These two diagrams show fertility and parental investment choices as a function of parent income taking child status as fixed under continuous and discrete fertility regimes.*

we have $n_i^e \in \mathbf{N}$ then introducing the condition of discrete n is not a binding constraint and thus does not alter the optimal choice. Because of the first order conditions, a slight perturbation of z around a value for which the discreteness constraint is not binding leaves the integer constrained n_i^e unchanged. By Lemma 2 we know further that n_i^e is stepwise increasing in z . Therefore intervals of z with n_i^e higher in the discrete n case alternate with intervals with n_i^e lower in the discrete n case. Since the marginal utility of b is decreasing in n it must be that b_i^e is greater in the discrete n case whenever n_i^e is lower in the discrete n case. Hence b_i^e in the discrete n case is alternating between being higher and lower than in the continuous n case.

Compare Figure 1.2.

1.6 Equilibrium Analysis

In the pure strategy symmetric Nash equilibria of this game, agents of different income groups compete for the status of their children and the poorer agents' threat to mimic may cause richer agents to choose inefficiently high levels of parental investment and forgo additional children. I denote the equilibrium choices of fertility n and parental investment b by a representative agent of income z_i by N_i and B_i , respectively.

It is clear upon reflection that the only channel by which equilibrium choices can move away from the privately optimal is that of the threat of loosing status in the second generation. This means that a parent agent can be kept from deviating away from an inefficiently high level of equilibrium parental investment B_i to a lower investment level because such a deviation would result in his children having a lower income than the children of other agents in his income group and thus being assigned a status lower than the equilibrium child status of his income group, which is denoted $s_{c,i}$. This reasoning also implies that equilibrium parental investment cannot be lower than the privately optimal investment given the equilibrium status, $b_i^e(s_{c,i})$; this is so because the children of an individual agent cannot lose status if their parent invests more in them. There is, however, an upper bound on how far B_i can move away from the privately optimal level of parental investment $b_i^e(s_{c,i})$. For this define \underline{u}_i as the utility level associated with the best deviation option of an agent with income z_i . We have $\underline{u}_i = \max \{V_i(z_i|n_i=0), \max_j V_i(z_i|\check{B}_j, s_{c,j})\}$ $j \neq i$ where $\check{B}_j = \max\{B_j, b_j^e(s_{c,j})\}$. The definition of \underline{u}_i reflects the deviation options of the agent: decide not to have children at all or choose a parental investment level that leads to a child status other than the equilibrium child status of his income group. In the latter case, conditional on targeting a certain child status, the agent may want to invest more than is necessary for his child to attain that certain status. He does not need to consider deviations targeting the equilibrium child status of his income group as we have established that $B_i \geq b_i^e(s_{c,i})$ so that these deviations would yield a lower total utility than the agent enjoys in equilibrium.

Let the upper bound on B_i be called \bar{B}_i .

Lemma 3 UPPER BOUND ON B_i GIVEN BY \bar{B}_i

Define $Q(b) = V(z_i|B_i=b) - \underline{u}_i$. Then we have:

$$\bar{B}_i = \begin{cases} z_i - \beta & \text{if } Q(z_i - \beta) > 0 \\ \text{solution to } Q(b) = 0 & \text{otherwise} \end{cases}$$

In equilibrium it holds that $B_i \leq \bar{B}_i$.

It can be shown that $\bar{B}_i > b_i^e(s_{c,i})$; it is increasing in $s_{c,i}$, and it is also increasing in z_i as long as the agent spends more on his children in total in the equilibrium

outcome than in the deviation outcome.

Proof: See Appendix.

Intuitively, equilibrium utility is falling in $B_i > b_i^e(s_{c,i})$, while the deviation options represented in \underline{u}_i are unaffected; so high equilibrium parental investment can only be supported up to the threshold \bar{B}_i .

It is clear that, unless incomes are vastly different, parent agents of a lower status may have an incentive to mimic parent agents of a higher status in terms of parental investment levels in order to secure a higher status for their children. To prevent this from happening, higher status parent agents have to raise their parental investment; they can do this because their higher income lowers their opportunity cost. The process is complicated in this model by the fact that higher parental status also raises the opportunity cost by increasing the marginal utility of parent consumption; income differences must therefore be big enough to allow the ‘parent income effect’ to dominate sufficiently the ‘parent status effect’.

Let the lowest level of parental investment by agents with income z_i which is incentive compatible be denoted by \underline{B}_i , where incentive compatibility is understood in the sense that neither do agents income $z_j < z_i$ want to deviate to $\langle B_i, s_{c,i} \rangle$ nor do agents of income z_i want to invest more in their children than \underline{B}_i . The derivation is as follows.

Lemma 4 LOWER BOUND ON B_i GIVEN BY \underline{B}_i

Define $\underline{B}_i = \max \{b_i^e(s_{c,i}), \{\hat{B}_{j,i}\}\}$ $j > i$ where $\hat{B}_{j,i}$ is the level of parental investment to which an agent from income group j would deviate to secure the status of $s_{c,i}$ for his child(ren); then it holds that $B_i \geq \underline{B}_i$.

Proof: By contradiction. Suppose $B_i < \underline{B}_i$, then agents from at least one income group $j \geq i$ will want to deviate and the putative equilibrium is destroyed.

□

It follows from the above Lemma in conjunction with Lemma 3 that levels of parental investment can only be supported in equilibrium on a certain closed

interval, we have $B_i \in [\underline{B}_i, \bar{B}_i]$ for all income groups i .

Note that $[\underline{B}_i, \bar{B}_i]$ depends on the values the B_j with $j \neq i$ take.

The second choice variable, the number of children, is only indirectly influenced by the competition for status which ensues when child status is endogenous.

Given the level of parental investment B_i , the choice of n does not affect the status of the agent's children. Hence, conditional on B_i , his best response to the other agents' strategy is the privately optimal strategy, which takes the status of children as given. And so in equilibrium we have $N_i = n_i^e(B_i, s_{c,i}) \forall i$.

We can now combine these findings in the following proposition.

Proposition 1 CHARACTERISATION OF SEPARATING EQUILIBRIA WITH EQUAL TREATMENT

An $|Z|$ -tuple of $\langle B_i, n_i^e(B_i, s_{c,i}) \rangle$ constitutes a symmetric pure strategy separating equilibrium with equal treatment of children iff given $\langle B_i \rangle$ we have $[\underline{B}_i, \bar{B}_i]$ nonempty $\forall z_i \in Z_p$ and $B_i \in [\underline{B}_i, \bar{B}_i]$.

Proof: The fact that $[\underline{B}_i, \bar{B}_i]$ nonempty given $\langle B_j \rangle$ with $j > i$ and $B_i \in [\underline{B}_i, \bar{B}_i] \forall z_i \in Z_p$ implies that, by Lemmata 3 and 4, the strategies $\langle B_i, n_i^e(B_i, s_{c,i}) \rangle$ are best responses to one another and thus constitute a symmetric Nash equilibrium in pure strategies under the assumption of equal treatment of children by each agent.

Conversely, if a such an equilibrium exists then the set of equilibrium strategies $\langle B_i, n_i^e(B_i, s_{c,i}) \rangle$ is incentive compatible and thus it must be that $\forall z_i \in Z$ $[\underline{B}_i, \bar{B}_i]$ nonempty given $\langle B_i \rangle$ and $B_i \in [\underline{B}_i, \bar{B}_i]$. \square

Note that the equilibrium need not be unique because potentially any $b \in [\underline{B}_i, \bar{B}_i]$ can be picked as B_i . Potentially, one cannot pick all of these values, because the condition $b \in [\underline{B}_i, \bar{B}_i]$ ensures incentive compatibility only for agents of income z_i , it does not take into account whether b renders some $[\underline{B}_j, \bar{B}_j]$ empty for some $j \neq i$ by changing $\hat{B}_{i,j}$ or \bar{u}_j .

Separating equilibria can more easily be sustained the greater the difference between the separating agents; in our case it holds that greater differences in

equilibrium child status and greater income gaps between income groups respectively raise the benefits and lower the costs of separating behaviour.

In particular, by making the difference between the income levels of agents of any two income groups, or ‘income gaps’, sufficiently wide, we can ensure the existence of separating equilibria.

Proposition 2 INCOME GAPS CAN ENSURE THE EXISTENCE OF SEPARATING EQUILIBRIA

For any \hat{S} , \exists a parent income distribution Z such that the condition in Proposition 1 is met and a separating equilibrium exist.

Proof: By induction. Set $z_{|Z_p|} > 0$ and $B_{|Z_p|} = b_{|Z_p|}^e(s_{c,|Z_p|})$ and $N_{|Z_p|} = n_{|Z_p|}^e(s_{c,|Z_p|})$. Let $i=|Z_p|-1$ and then since $\frac{\partial b_i^e(s_{c,i})}{\partial z_i} > 0 \exists$ a threshold income level \hat{z}_i (alternatively, a threshold income gap $\widehat{z_i - z_{i-1}}$) above which $b_i^e(s_{c,i}) \geq \max_j \{\hat{B}_{j,i}\}$ where $j > i$ and thus $\bar{B}_i > b_i^e(s_{c,i}) = \underline{B}_i \Rightarrow [\underline{B}_i, \bar{B}_i]$ nonempty; for $i = |Z| - 1$ choose $z_i \geq \hat{z}_i$ and some $B_i \in [\underline{B}_i, \bar{B}_i]$ and $N_i = n_i^e(B_i, s_{c,i})$. Repeat process for $i-1$ until $i = 1$. \square

Note that the reverse is not true, i.e. we need not find a set of stati \hat{S} to fit any set of income levels Z such that an equilibrium exists. The reason is that the aforementioned ‘parent status effect’ dominates the ‘parent income effect’ for very small income differences.

1.7 Fertility Behaviour in Equilibrium

The equilibrium fertility behaviour is influenced only indirectly by the competition for status. As has been shown above equilibrium parental investment is greater or equal to the level an agent would choose if the status of his child was exogenously fixed at the equilibrium child status of his income group. Since the equilibrium number of children is equal to the efficient number given the equilibrium parental investment level and the efficient number of children is falling in the chosen level of parental investment into each child (which in equilibrium

is the equilibrium parental investment level) - as established in Lemma 2 - the equilibrium number of children is less than or equal to the privately efficient number of children.

This is formalised in the following proposition.

Proposition 3 FERTILITY EQUAL TO OR LOWER THAN PRIVATELY OPTIMAL

Define $Q(b|k) = V(z_i|b, n_i^e(s_{c,i})-k) - V(z_i|b, n_i^e(s_{c,i})-k+1)$ with $k \in \{1, 2, \dots, n_i^e(s_{c,i})\}$.

Then define $\tilde{b}_{i,k}$ by the solution to $Q(b|k) = 0$. It can be shown that $\tilde{b}_{i,k}$ is unique, greater than $b_i^e(s_{c,i})$ and increasing in k .

If $\bar{B}_i > \tilde{b}_{i,k}$ then by setting B_i such that $\min\{\bar{B}_i, \tilde{b}_{i,k+1}\} \geq B_i > \tilde{b}_{i,k}$ agents of income z_i will choose $N_i = n_i^e(s_{c,i})-k$.

Proof: See Appendix.

If for example the difference in equilibrium child status between agents of income z_i and z_{i-1} is big - that is $s_{c,i} - s_{c,i-1}$ is big - then agents of income z_i are willing to accept high inefficiency in their equilibrium parental investment, in other words \bar{B}_i is high. It may be so high that $B_i = \bar{B}_i$ implies a reduction in fertility compared to the efficient choice.

This effect of status competition need not affect all income groups to the same degree. If for instance status competition is stronger for income groups with higher income, then the positive intrinsic relationship between income and number of children may be reversed.

This is best shown in an example.

Example 3 A SIMPLE NON-MONOTONIC FERTILITY EQUILIBRIUM

Set $Z_p = \{z_1, z_2, z_3\}$ and $\hat{S} = \{s_1, s_2, s_3\}$. Think of s_2 as substantially larger than s_3 . Suppose that z_1 is so high that both $b_1^e(s_1) \geq z_2 - \beta$ and $n_1^e(s_1) > 2$ and suppose that $B_1 = b_1^e(s_1)$. Suppose further that z_3 and s_3 are such that $n_3^e(s_3) = 2$ and that s_2 is such that $\hat{B}_{3,2} = z_3 - \beta$, i.e. agents of income group 3 would be willing to spend all their income on a single child if that would secure the parental status of income group 2 for that child. Suppose that the last relationship just so holds and that $B_3 = b_3^e(s_3)$, then:

$$\alpha \cdot u(f(z_3 - \beta)) \cdot s_2 = u(z_3 - 2(b_3^e(s_3) + \beta)) \cdot s_3 + \alpha \cdot 2^{1-\epsilon} u(f(b_3^e(s_3))) \cdot s_3 \quad (\text{I})$$

Suppose for a moment that $z_2 \approx z_3$, then the left hand side (LHS) of (I) would be the same for income group 2 with a putative $B_2 = z_3 - \beta$ while the right hand side (RHS) would be higher due to $s_{p,2} = s_2 > s_3 = s_{p,3}$. Agents of income z_2 would thus want to deviate from $B_2 = z_3 - \beta$ to B_3 and so the putative separating equilibrium is destroyed at $z_2 \approx z_3$.

If, in our mind, we slowly raise z_2 while holding on to the putative equilibrium with $B_2 = z_3 - \beta$, then the LHS of (I) will initially grow faster than the RHS because $u(\cdot)$ is concave and additional income will go entirely into parent consumption on the LHS; this is true at least up to the point where $B_2 + \beta = N_2^{dev}(B_2^{dev} + \beta)$ where $B_2^{dev} = \max\{B_3, b_2^e(s_3)\}$ and $N_2^{dev} = n_2^e(B_2^{dev})$ - up to this point parent consumption is lower under the LHS choice and thus the marginal utility of income is higher on the LHS. At that point the difference of LHS - RHS is $\alpha \cdot u(f(z_3 - \beta)) \cdot s_2 - \alpha \cdot N_2^{dev^{1-\epsilon}} u(f(B_2^{dev})) \cdot s_3$; if s_2 is sufficiently larger than s_3 this difference is ≥ 0 . Then we have that $[\underline{B}_2, \bar{B}_2]$ nonempty and thus the putative equilibrium exists.

If we set z_2 , s_2 and s_3 such that this situation holds, then the result is the following non-monotonic fertility behaviour:

	B_i	N_i
income group 1	$b_1^e(s_1)$	> 2
income group 2	\underline{B}_2	1
income group 3	$b_3^e(s_3)$	2

This example paints an abstract picture of a society in which fertility is a U-shaped function of parent income. While households at the lower and upper end of the income spectrum have relatively many children - the poor due to the absence of competition for their status and the rich due to a high enough income gap to fend off competition easily - the average income household, which we can perhaps call middle class, struggles to defend its status in the next generation and forgoes additional children in the process.

As a corollary, if we think of economic development as raising the purchasing power of the poor as a percentage of the purchasing power of the middle class - which can be motivated for instance in an anecdotic manner by the observation that the average number of household helps in a middle class family declines

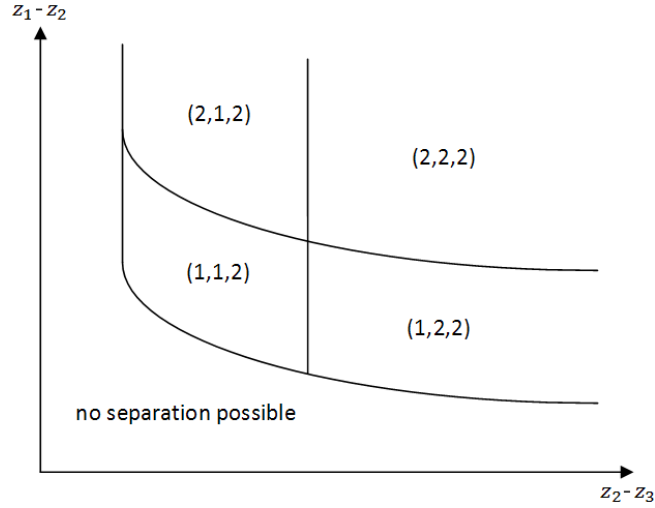


Figure 1.3: *Fertility outcome (N_1, N_2, N_3) as a function of income gaps. I assume that agents coordinate on $B_i = \underline{B}_i$ and that $n_3^e = 2$. Lacking competition from below, the lowest income parent agents thus have $B_3 = \underline{B}_3 = b_3^e$. Example 3 described the case in the top left, i.e. low income differences between low and medium income parents and high income difference between medium and high income parents.*

as a country develops - then, by making the income distribution more equal, economic development may bring about a demographic transition partly through the channel of increased competition for status.

Proposition 3 does not, however, necessarily imply a U-shaped fertility function. As Figure 1.3 shows for a chosen parameter set, fertility functions depend on the income gaps. We get the U-shape when competition is fierce for the medium status and not for the top status.

Note that the income gap between high and medium income agents needs to be higher when the gap between medium and low income agents is low because then equilibrium utility of the medium income agents is depressed by $\underline{B}_2 > b_2^e$ and so medium income agents are willing to spend more for successful mimicking ($\hat{B}_{2,1}$ is higher).

The next example uncovers an interesting possible result of the ‘parent status effect’; it shows that the above non-monotonicity result may be sharpened to the case where higher income agents choose to remain childless. This is a surprising result for preferences that make both number of children and parental investment in each child intrinsically normal goods.

Example 4 A CHILDLESS MIDDLE-CLASS

Set $Z_p = \{z_1, z_2, z_3, z_4\}$ and $\hat{S} = \{s_1, s_2, s_3\}$; this way income groups 3 and 4 will share the parent status of s_3 while groups 1 and 2 will enjoy the higher parent status of s_1 and s_2 , respectively. Suppose that z_1 is so high that both $b_1^e(s_1) \geq z_2 - \beta$ and $n_1^e(s_1) > 2$ and that $B_1 = b_1^e(s_1)$. Suppose further that $z_2 \approx z_3 \approx z_4$ and that these incomes are so low that income group 4 is about indifferent between having and not having a child:

$$u(z_4) \cdot s_3 \approx u(z_4 - b_4^e(s_3) - \beta) \cdot s_3 + \alpha \cdot u(f(b_4^e(s_3))) \cdot s_3 \quad (\text{II})$$

Then $N_2 = 0$. To see this, suppose that it is true; then the child of z_3 will enjoy status s_2 and we can set $B_3 = \underline{B}_3 > b_3^e(s_2) > b_3^e(s_3)$. Income group 3 will defend the status of its children against group 4 since parental status is the same and group 3 has a (small) income advantage. Income group 2 will be content not having children: a child status of s_1 is unattainable while, when aiming to let a child have status s_2 by mimicking income group 3, equation (II) holds with s_3 changed to s_2 and so an agent of income group 2 will not be willing to raise parental investment from $b_2^e(s_2) \approx b_3^e(s_2)$ to B_3 ; aiming for a child status of s_3 by mimicking income group 4 is not a profitable deviation either as the LHS of (II) will be larger than the RHS if only parental status is changed from s_3 to s_2 . The result can be summarised in the following table:

	$s_{p,i}$	$s_{c,i}$	B_i	N_i
income group 1	s_1	s_1	$b_1^e(s_1)$	> 2
income group 2	s_2	—	0	0
income group 3	s_3	s_2	$B_3 > b_3^e(s_2)$	1
income group 4	s_3	s_3	$b_4^e(s_3)$	1

The intuition is that the parent status of the middle class agents is high relative to the lower income groups while the income gap is small, so that the opportunity cost of defending the status for a child is higher than for parents of lower status.

One may think of the parent status effect as biasing the preferences of the agent towards children of higher status; it makes agents of higher status pickier about

their children. Loosely speaking, the decision is then less about having or not having children but about having highly successful children or no children.

A general condition for childlessness of this form is given in the following proposition.

Proposition 4 SOME INCOME GROUPS MAY CHOOSE TO REMAIN CHILDLESS
*Suppose that given $N_i = 0$ and $\langle B_j \rangle$ we have $[\underline{B}_j, \bar{B}_j]$ with $j \neq i$ nonempty and $B_j \in [\underline{B}_j, \bar{B}_j] \forall j$; suppose further that $u(z_i) \cdot s_{p,i} \geq \max_j V_i(z_i | B_j^{dev}, s_{c,j})$ with $B_j^{dev} = \max \{B_j, b_i^e(s_{c,j})\}$ for $j \neq i$.
Then \exists a (nonstandard) separating equilibrium in which $N_i = 0$.*

Proof: By construction. Taking $N_i = 0$ as given a separating equilibrium exists by Proposition 1. $N_i = 0$ is a best response by agents of income group i because there is no deviation yielding a higher utility than $V_i(z_i | N_i = 0)$. \square

1.8 Discussion

This paper has shown that separating equilibria which feature relative concerns in a multi-generational context may exist; and by Proposition 3 they may exhibit both inefficiently high levels of parental investment in children and inefficiently low numbers of children (from the point of view of the parents).

These effects need not be uniform over different income groups; as in Example 3, equilibrium fertility can be non-monotonic in income even though it is increasing in income when status is awarded exogenously. This contributes to our understanding of observed patterns of fertility in developed countries which show a high concentration of single child families in what might be termed the middle classes.

The equilibrium forces stemming from the competition for status in future generations are powerful; this is possibly best exemplified by Proposition 4 and Example 4 which document that, in this competitive environment, higher income groups may even find it not worthwhile to have children at all. The intuition of

voluntary childlessness which this paper develops builds on two elements: one is that the social aspirations of parents of lower status may make the status of children very costly; the other is that agents who themselves enjoy a comparatively high status have thus higher opportunity cost when deciding on the investment level of children. These elements together imply that potential parents who have achieved comparatively high status for themselves may choose not to have children because they value only high status children and these are made excessively costly by parents of lower status. They opt out of the game because their status has made them picky.

In the analysis I have been agnostic towards equilibrium selection. In real life, however, where generations overlap and ‘generation’ is a more gliding concept, the behaviour of earlier generations provides a natural focal point for equilibria. From this follows that once a society has coordinated on a low fertility equilibrium, e.g. due to a relative increase in incomes of the poorer income groups, this equilibrium becomes very stable. The model of this paper hence suggests that, unless some great change is inflicted upon the societies of most developed countries, their low fertility is permanent.

Endogenous status in this setup brings about pareto losses²¹; the equilibrium thus asks for correction by government intervention. A suitable system of Pigouvian taxes and subsidies may provide remedy. Societies may find it more effective, however, to try to make parent induced social mobility a thing of the past altogether by, as mentioned above, socialising parental investment; this would render child status exogenous to the parent agent and the externality would thus disappear. Yet whether this advantage outweighs the cost of potential incentive side effects would have to be subject of a more thorough study. The repeated finding of the literature on relative concerns that greater economic equality may not be welfare enhancing because of increased competition for status is present in this model as well. The measures to remedy the mentioned pareto losses therefore cannot be seen in isolation from the measures so-called welfare states undertake in order to reduce inequality of incomes.

Future work may carefully investigate the recent finding of an empirical J-

²¹I am referring to the parent generation only; the child generation is contentious because it is e.g. difficult to agree on how to treat the utility of unborn agents.

curve of fertility as a function of human development²² which may be speculated to be indicative of greater earnings inequality of high skilled vs. low skilled labour and/or decreased social mobility.

Finally, what would be this paper's answer to the advice seeking couple quoted at the beginning? It would recommend not to have another child and to allow their children to be as "demanding" as their peers. Even though the couple may privately lament their small family, if they feel like the agents of the model that is the best they can do.

²²Compare Myrskylä et al. [2009].

Chapter 2

Relative Concerns and Primogeniture

2.1 Introduction

Two hundred years ago, differential treatment was deeply engrained in many human societies: family farms in many regions were bequeathed to the eldest (or youngest) son; noble titles and estates were passed from one person to the next in line. One hundred years ago, very few women went to university showcasing a vastly differential treatment based on gender.

Why then do people in advanced societies today invest roughly equal amounts of money and time into each of their children; why has differential treatment of children all but disappeared?

While changes in social norms and the values people hold provide an obvious answer, this paper proposes that the underlying reason is that faster technological change as well as an increase in the economic importance of human capital have made the returns to parental investment into children riskier. Today, the position a person occupies in the income distribution and social hierarchy arguably depends more on his talent and luck than in the past. From the point of view of the parent this means less influence over the fate of his or her child. The parent can induce less differences among his children and the fact that his children are thus more equal in expectation implies, as shall be shown, that

favouring one or a group of children over the rest becomes less attractive.

The argument is developed in the context of a two generational model of altruism-based endogenous fertility in which agents simultaneously choose the (continuous) number of children to have and how much of their income to spend on parental investments into each of the children where parental investment into a child determines the future income of that child¹. Agents exhibit relative concerns, they foresee that the wellbeing of their children depends not only on their absolute consumption but also on their relative standing in society which is represented in the model by a cardinal rank that is assigned according to the income level of the agent.

Differential treatment of children arises in this model if two conditions are met: firstly, child wellbeing must over the relevant interval be a concave function of parental investment, so that the child type offering the highest marginal return has a lower investment level than the highest investment level type; secondly, parental affection towards each individual child must be decreasing in the total number of children of the agent (so that each additional child ‘dilutes’ the parent’s affection towards his siblings). Facing a trade-off between lower investment level, high marginal return, high dilution type children and high investment level, lower marginal return, lower dilution type children, the agent may choose a portfolio of both child types, and hence differential treatment, if dilution works in certain ways (which are explored below).

The general intuition of differential treatment in the model of this paper is difficult to provide, it depends on the nonlinearity of both the return on parental investment and of parental affection towards each child. It is clearer in an example: suppose dilution is convex such that it does not matter for low numbers of children. A relatively poor agent will thus choose to have a certain (low) number of the lower investment level, high marginal return children. A relatively rich agent would want to invest more into his children than the poor agent because the marginal utility of his consumption is lower. Adding more lower investment level children to the number chosen by the poor agent, however, may not be the optimal solution because dilution is assumed to become severe for higher

¹This paper essentially takes the model of Chapter 1 and generalises it by allowing agents to treat their children differentially. Further departures include the change of the rank assigned to very low out of equilibrium child incomes and the relaxation of the integer constraint on fertility and the introduction of uncertainty into the assignment of ranks.

numbers of children. Switching all children to the expensive type may not be optimal either because the gain in lower dilution is low compared to the loss in total child wellbeing as dilution is low for low numbers of children. In order to participate without too much loss (due to dilution) in the wellbeing of the high marginal return children while investing more in his children than the poor agent, he may therefore complement lower investment level children with (only a few) expensive children that still offer a higher return than parental consumption.

Uncertainty, or parental investment risk, is captured in the model by having social rank assigned to income levels in a noisy way (i.e. even a high income child may end up with a low rank and a low income child may be awarded a high rank; this setup can be interpreted as a reduced form model of idiosyncratic or ‘talent risk’ of children). As it is high investment level child types whose expected return is lowered by more uncertainty and low investment level child types whose expected return is raised, it follows firstly that any convexity of expected status given child income is weakened by higher uncertainty. In equilibrium, agents secondly respond to this by raising per child investment in low investment child types and decreasing per child investment in high investment child types, the effect of which is that high investment child types offer less of a protection against dilution. Both these forces tilt the trade-off between medium investment, high marginal return children and high investment, low dilution children in favour of the former. Eventually, the solution to this trade-off is a corner solution and then differential treatment disappears.

In the next section I review the received literature on differential treatment of children; in the following I present the argument of the paper in a simple example while the ensuing sections introduce and analyse the model and the final section concludes.

2.2 Related Literature

Differential treatment of children is predominantly studied in its extreme form, namely ‘unigeniture’ or the inheritance of most of the parents’ wealth by just one child. Historically, this was usually the first-born son and hence authors

often refer to the better known word of ‘primogeniture’, which is unigeniture with the first born as the beneficiary.

The overwhelming majority of the economic literature on primogeniture (implicitly) assumes that parents intrinsically care the same way about all their children and looks for differences in the children to explain the phenomenon of differential treatment and primogeniture². These differential characteristics can either be exogenously given or endogenously given (via increasing returns to scale).

The literature focusing on exogenously given differences falls in two parts, one modelling a world of functioning capital markets and one modelling a world without.

While not necessarily addressing the extreme form of primogeniture, authors such as Becker and Tomes [1979] or Sheshinski and Weiss [1982] have developed models in which parents care for the well-being of their children and treat them differentially for the following reason: with differences in child ability outside the control of the parents, different levels of parental investment equalise marginal returns of parental investment. By differential treatment the parent thus capitalises on the different investment opportunities offered by his children. Depending on whether his investment is more substituting or complementing he thereby compensates the less able ones or reinforces the success of the more able ones.

In a world without capital markets, Grieco and Ziebarth [2010] model primogeniture as an intra-family insurance device: if the future income of heirs is subject to a random shock and heirs do not care about each other intrinsically, then the parent may buy brotherly and sisterly concern from one child in return for a greater share of the inheritance.

The strand of literature advancing endogenous differences goes back to Smith [1776] who remarked that “[i]n those disorderly times, every landlord was a sort

²For an exception, see for instance Faith [2001] and Faith et al. [2008] who interpret inheritance rules as a parental response to children seeking more of the total bequest than their siblings. In particular, Faith [2001] argues that, equal treatment being optimal because it minimises wasteful rent seeking behaviour of the children, primogeniture was upheld by the church in an effort to “maximise the value of its monopoly power over a whole host of social services”.

of petty prince. [...] The security of the landed estate depended [...] upon its greatness. To divide it was to ruin it [...]. Hence the origin of the right of primogeniture, and of what is called lineal succession.” (Book III, Chapter 2). This is the sort of political increasing returns we shall encounter further below.

Drawing on anthropological literature such as Nakane [1967] and Cole and Wolf [1974], Chu [1991] reinterprets Adam Smith’s observation by letting the objective of the family head be to minimise the probability of lineal (dynastic) extinction. His main interest being in the interplay of primogeniture and income inequality, he only notes in passing that the reason for primogeniture in his model is increasing returns to parental investment - be it in the survival probability into adulthood of a child or in the future wealth of a child. He further argues that these reasons play out better in the absence of perfect capital markets.

Bergstrom [1994] works out the evolutionary fitness of different roles in a stratified society which restricts polygyny to wealthy males in an effort to throw light on how fitness maximising preferences may sustain such societal setups. With constant costs of additional children, primogeniture emerges as a response to increasing political returns to wealth: great fortunes are more easily defended when property rights may be disputed and so the return on investments increases in the scale of total investment.

In light of Bergstrom’s analysis, the urge to perpetuate the family line by a legitimate male heir as in Chu [1991] can be reinterpreted as the care taken to ensure that one’s wealth (as the key to great reproductive success) passes on to a person with whom one shares many genes.

In a simpler way, increasing returns are also invoked by Bertocchi [2006] who models primogeniture of landed estates as the rational reaction to an indivisibility constraint due to minimum efficient scales of production.

And drawing on a vast array of historical accounts, Hrdy and Judge [1993] argue that differential treatment of children is a response to scarcity of arable land in a marriage market setup that bars men below a certain minimum wealth from reproduction. While increasing returns are obvious in this argument, they may be less so in a second argument: they point to the fact, noted by biologists such as Gillespie [1977], that, given an equal expected number of offspring, types with the lower variance of reproductive outcomes fare better in the long run.

Concentrating one's wealth on a single child with fairly predictable reproductive success would thus be superior to dividing the wealth among more children thus placing many smaller bets. But this again, is simply increasing returns in the form of lower variance per unit invested as total investment grows.

The present paper adds to the strand of literature that explores endogenous differences of children. While increasing returns to parental investment are present in the form higher investment leading to higher expected social rank of the child, it is shown that for differential treatment to arise the altruism of a parent towards his children must depend on his total fertility in subtle ways. Its main contribution to the literature is to show that the disappearance of differential treatment in modern societies may be due to greater uncertainty over the payoff of parental investment into children.

2.3 Illustrating Example

In this example I consider only the incentives faced by an individual agent who has to decide how many children to have and how much to invest in each child. Let his choice of how much to invest be constrained such that he may choose only from three investment levels. I shall refer to a child endowed with the highest allowed investment level as a type 1 child, a child with the second-highest allowed investment level as a type 2 child and a child with the lowest allowed investment level as a type 3 child. This constraint shall be motivated in the next section, for this example I take it as given.

Let the well-being of a child be given by a formulation following Frank [1985b]:

$$u(q \cdot b_j) \cdot S_c(q \cdot b_j) \tag{2.1}$$

Where b_j is the parental investment level of a type j child with $j \in \{1, 2, 3\}$ and $q > 0$ is a parental investment productivity parameter so that given the agent invests b_j into a child the income of that child is going to be $q \cdot b_j$. With $u(\cdot)$ a standard concave consumption utility function as well as $S_c(\cdot)$ the rank functions mapping child income to child social rank, the instantaneous utility or well-being of a child is thus given by the product of its instantaneous consumption utility and social rank.

I let the rank function S_c be a step function which, for the three child types, approximates a convex function. The interplay of consumption utility, social rank and well-being is illustrated in Figure 2.1.

I let the agent have exogenous income and be altruistic towards his children. And I begin by abstracting from the concept of dilution. Following the dynastic utility function of Becker and Barro [1988] adjusted for two generations, the maximisation problem of the agent can then be written as:

$$\max_{n_j} \left\{ u\left(z - \sum_j n_j(b_j + \beta)\right) \cdot S_p(z) + \alpha \sum_j n_j \cdot u(q \cdot b_j) \cdot S_c(q \cdot b_j) \right\} \quad (2.2)$$

Where $z > 0$ is the income of the agent, n_j is the number of type j children that the agent chooses to have, $S_p(\cdot)$ is the rank function in the parent generation, $\alpha \in (0, 1)$ is a altruism parameter that measures how much the agent cares for the wellbeing of his children compared to his own instantaneous utility and $\beta > 0$ is a fixed cost of raising a child.

The solution to this maximisation problem is to demand only the child type that offers the highest marginal return on parental investment or, equivalently, offers the highest return ratio $\frac{u(q \cdot b_j) S_c(q \cdot b_j)}{b_j + \beta}$. In the exemplary setup introduced in Figure 2.1 this would be child type 2. This is shown in Figure 2.2: of the rays going from the origin through the child well-being and investment points the one associated with child type 2 has the steepest slope.

Given these preferences, there is thus no differential treatment of children.

Next, I introduce the notion of dilution which means that the affection the agent feels towards each of his children is decreasing in the total number of his children. Formally, we can write $a(\cdot)$ for a strictly decreasing and positive dilution function and then have the agent face:

$$\max_{n_j} \left\{ u\left(z - \sum_j n_j(b_j + \beta)\right) \cdot S_p(z) + \alpha \cdot a\left(\sum_j n_j\right) \sum_j n_j \cdot u(q \cdot b_j) \cdot S_c(q \cdot b_j) \right\} \quad (2.3)$$

The effect of dilution is that each additional child imposes a ‘between children’ externality by lowering the altruistic feeling that the parent carries towards the other siblings. Dilution makes having many children less attractive. As for the choice of child types this means that *ceteris paribus* child types with higher

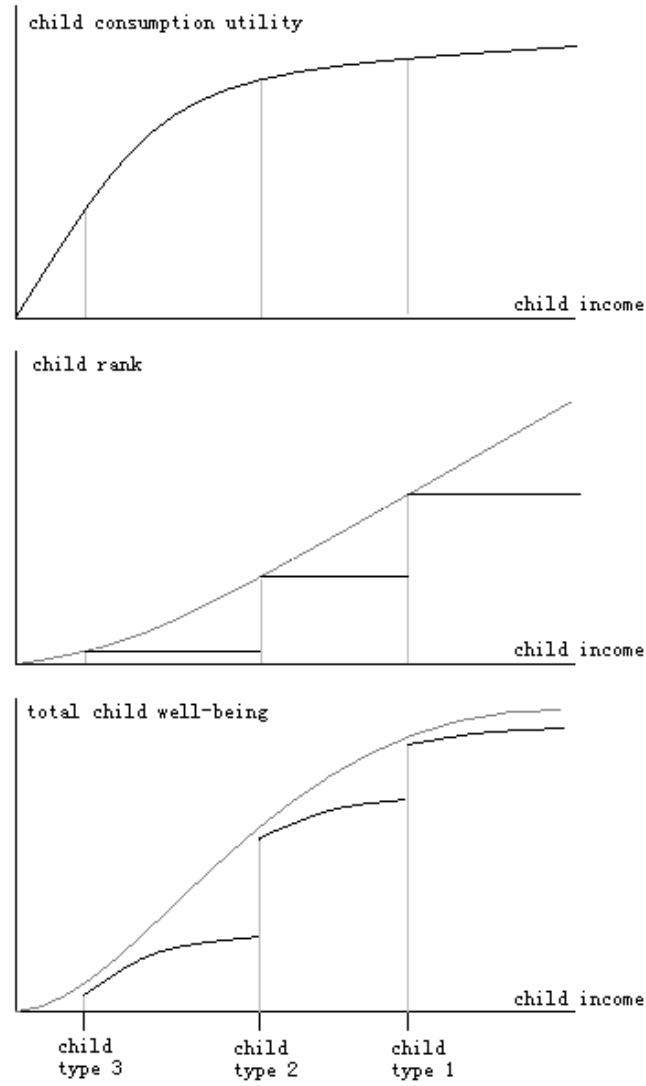


Figure 2.1: *These diagrams show how decreasing marginal consumption utility together with a rank function that approximates a convex function generate returns to parental investment that are convex for low investment levels and concave for high investment levels.*

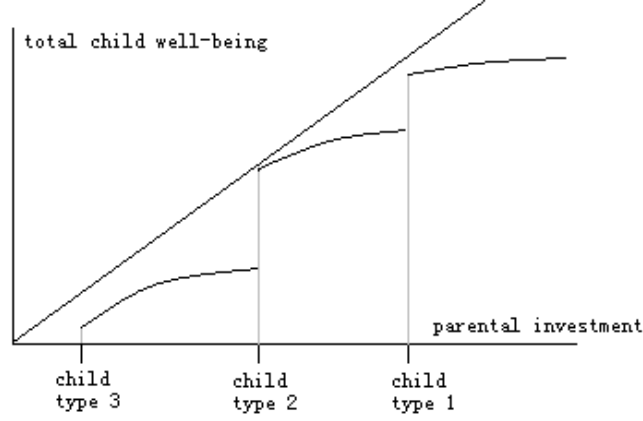


Figure 2.2: *This diagram shows that child type 2 offers the highest marginal return on parental investment.*

investment level are preferred: being more expensive they translate into fewer children per unit of investment and thus dilute the parent's altruism less.

Suppose now that the dilution function has a shape as shown in Figure 2.3. Suppose further that the agent will optimally invest a combined amount x into all his children. The agent can have arrived at this choice by only three ways:

- (a) demand $n_C = \frac{x}{b_2 + \beta}$ type 2 children which offer the highest return ratio,
- (b) demand $n_A = \frac{x}{b_1 + \beta}$ type 1 children which offer a lower return ratio but are more expensive and hence would result in a lower total number of children, which means $n_A < n_C$, so that the agent would value his children's well-being more,
- (c) demand both type 1 and type 2 children so that the total number of children is at, say, n_B and the average return ratio is between those of the two child types. Even though we cannot say which of the three options the agent has chosen without specifying $u(\cdot)$, $a(\cdot)$ and the values of the social ranks more precisely, it is clear that for some specifications the agent's choice would be (c) and with that we have differential treatment of children.

It is already clear at this point that if child type 1 (with the highest level of parental investment) had offered the highest return ratio then the agent would have only demanded type 1 children (option (b)) as in that case both return ratio effect and dilution effect would have pulled in the same direction.

Uncertainty removes differential treatment by lowering the dispersion of the income levels of child types. This means that dilution plays less of role; and if its role is weak then we are effectively in a world without dilution as given in the

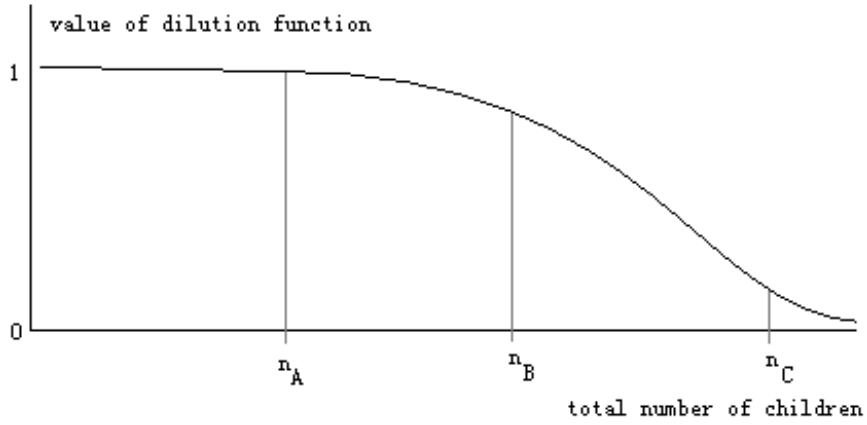


Figure 2.3: A dilution function $a(\cdot)$ is shown which decreases very slowly up to a threshold total number of children and then falls very quickly. Three possible total numbers of children are marked off as n_A , n_B and n_C .

preferences of Equation (2.2) which has been shown not to allow for differential treatment of children.

To see how this works we need an understanding of how the child types the agent chooses from come about in equilibrium. This is developed in the following sections.

2.4 Model Setup

The aim of the model to be built is to provide insight into the rationale people may have when they choose, under the presumption that relative comparisons matter to them, how many children to have and how to prepare these children for a productive life in society. Consequently, below I shall put agents in a setting in which they have only this choice to make and in which parental investment in a child determines the (expected) social position, or rank, this child is going to occupy.

Imagine a world of two generations where parent agents live for one period in which they receive an exogenously given amount of income and decide how to spend it on their own consumption and on investments in their children, the number of which they choose. The population of parent agents to be studied is of

a certain measure and is divided into a finite number of groups, each of positive measure. Within each group income levels of agents are homogenous. Groups differ in the associated income levels and may differ in size. An individual agent is of measure 0. Call the set of income levels that form the support of the parent income distribution Z_p with elements $z_{p,1} > z_{p,2} > \dots$

In the pure strategy symmetric Nash equilibria that I concentrate on the discrete nature of the income distribution in the parent generation is preserved in the child generation no matter what strategies the parent agents choose. I call the support of the income distribution in the child generation Z_c with elements $z_{c,1} > z_{c,2} > \dots$

Imagine further that, as introduced in the example above, agents care about their relative standing in society, their status or social rank, and that this rank is awarded to agents according to their income such that agents of higher income enjoy a higher (expected) status. Imagine finally that parental investment in a child determines the income of that child.

Central to our problem is therefore the mapping of agent income to agent rank (or status). This mapping is captured by the status function $S_x(\cdot)$ where $x \in \{p, c\}$ indicates the generation it is referring to. It is to be thought of as a step function which takes steps at income thresholds called $\hat{z}_{x,j}$ (again subscripted for the generation) which are taken from the set \hat{Z}_x and ordered such that $\hat{z}_{x,j} > \hat{z}_{x,j+1}$. The values that $S_x(\cdot)$ can take are recorded in the discrete set of ranks R with element r_j where the ordering is again such that $r_j > r_{j+1}$; to simplify the exposition below I assume that $|R| = |Z_p|^3$.

I introduce a stochastic element in the assignment of ranks to income levels. In particular, children enter a lottery over the ranks recorded in R where their odds are a function of their income. The motivation for this setup is that it captures the fact that parents do not have perfect control over the social success of their children, much depends on luck and ability. While it may appear more natural to model this by letting child income be stochastic, letting their rank be stochastic simplifies the analysis without compromising on the intuition; one may think of this combination of deterministic child income and stochastic child ranks as the utility of children as a function of parental investment in its reduced form just as the mentioned lottery can be interpreted as the reduced form of a

³This stepwise function $S(\cdot)$ is similar to the category reporting case of Harbaugh [1998].

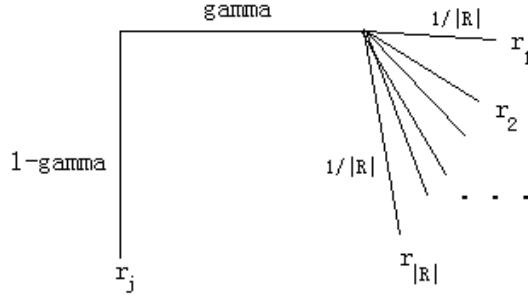


Figure 2.4: *This diagram shows the lottery a child enters whose income is greater than or equal to the income threshold $\hat{z}_{c,j}$ and lower than the income threshold $\hat{z}_{c,j-1}$. If the income of the child is greater than or equal to the highest income threshold, then $j = 1$. (The diagram presupposes that there are at least $|R|$ income groups in the child generation, otherwise replace $|R|$ with $|Z_c|$.)*

imperfectly discriminating or noisy contest for child social rank which is based on child income.

To be precise, imagine the following procedure: a child with income level z_c where $\hat{z}_{c,j-1} > z_c \geq \hat{z}_{c,j}$ enters a lottery in which with probability $(1 - \gamma)$ it is assigned a rank of r_j and with probability γ it enters a second lottery which assigns a rank by a random draw from the set R . Let the expected rank of such a child be denoted by \tilde{r}_j ; and let \tilde{R} be the set of all expected ranks \tilde{r}_j with j running from 1 to $|R|$. As a special case, I set rank equal to 0 if child income is below the lowest threshold, i.e. $S_c(z|z < \hat{z}_{c,|\hat{Z}_c|}) = 0$. Compare Figures 2.4 and 2.5 and note that by construction $\tilde{r}_j > \tilde{r}_{j+1}$.

γ can be thought of as the degree of uncertainty over the success of a child (from the point of view of the parent agent) and we can note that for $\gamma=0$ the assignment of ranks becomes deterministic while for $\gamma=1$ rank assignment is completely random so that \tilde{r} is unresponsive to the child's income level as long as it is $\geq \hat{z}_{c,|\hat{Z}_c|}$.

To simplify the exposition, I let ranks of parent agents be assigned in a deterministic manner, i.e. as if $\gamma=0$, so that there is uncertainty only over child ranks. While the relative value of rank in society is taken to be determined by technology and social institutions, so that R is common to both generations, I suppose that the income thresholds that influence which rank an agent with a certain income occupies change faster; therefore \hat{Z}_p need not be equal to \hat{Z}_c .

We have assumed above that the income distribution of parent agents is discrete and called its support Z_p . We now let the income thresholds of the parent gen-

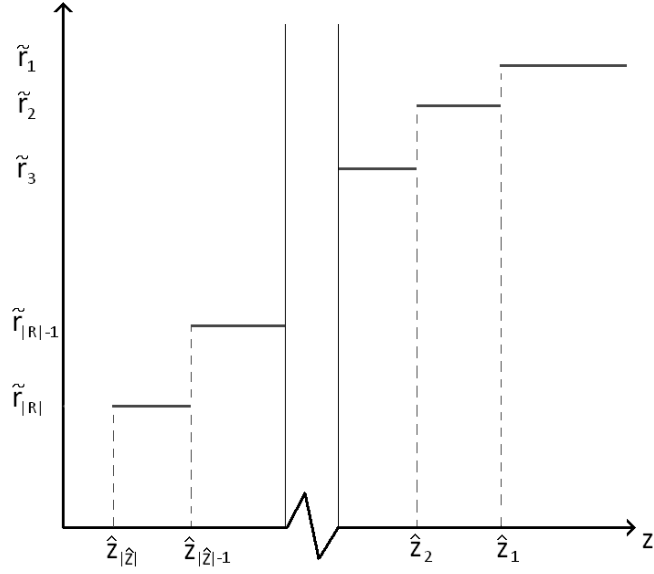


Figure 2.5: Graphical representation of the rank function S given the set of income thresholds \hat{Z} and set of expected ranks \tilde{R} .

eration be given by the observed income levels, i.e. $\hat{Z}_p = Z_p$. This means that the richest parent agents (with income $z_{p,1}$) enjoy the highest rank (r_1) and the second richest the second highest rank ($z_{p,2}$ and r_2) and so on until $z_{p,|Z_p|}$ and $r_{|R|}$ (we have assumed that $|Z_p| = |R|$).

Only in the child generation are the income thresholds endogenous. In pure strategy symmetric equilibria the distribution of child incomes will be discrete so I can let the $|R|$ highest observed child income levels (or all income levels if $|Z_c| \leq |R|$) be the set of income thresholds \hat{Z}_c . In other words, the richest among the children (with income level $z_{c,1}$) will enjoy an expected rank of \tilde{r}_1 , the second richest an expected rank of \tilde{r}_2 and so on until $\tilde{r}_{|R|}$ (or $\tilde{r}_{|Z_c|}$). Should there be more income groups than $|R|$ in the child generation, then the $|Z_c| - |R|$ poorest will have a rank of value 0.

Slightly generalising the maximisation problem in Equation (2.3) of the example above, we can summarise the decision problem faced by a parent agent with income $z_{p,i}$ in the following way:

$$\begin{aligned} & \max_{b_{i,j}, n_{i,j}} \left\{ u \left(z_{p,i} - \sum_j n_{i,j} (b_{i,j} + \beta) \right) \cdot S_p(z_i) \right. \\ & \left. + \alpha \cdot a \left(\sum_j n_{i,j} \right) \sum_j n_{i,j} u \left(f(b_{i,j}; z_{p,i}) \right) \cdot E \left[S_c \left(f(b_{i,j}; z_{p,i}) \right) \right] \right\} \quad (2.4) \end{aligned}$$

Note that in contrast to (2.3), the agent chooses both the parental investment levels and the fertility level of the child types he wishes to have: the $b_{i,j}$'s stand for the chosen parental investment levels of his children and each $n_{i,j}$ stands for the number of children with the parental investment of $b_{i,j}$ the agent chooses to have.

As above, apart from parental investment each child also costs the agent a certain fixed amount $\beta > 0$; $u(\cdot)$ is a strictly concave and twice differentiable utility function that transforms consumption into instantaneous absolute consumption utility; $S(\cdot)$ maps an agent's income to his rank in society and is subscripted by the generation; $1 > \alpha > 0$ is the altruism parameter that measures how much an agent values the well-being of his (single) child compared to how much he values his own consumption; and $a(n_i) > 0$ with $a(1)=1$ is a strictly decreasing function of the number of children that captures the dilutive effect of additional children on the altruistic feelings of the parent towards each individual child. $f(b_{i,j}; z_i)$ generalises $q \cdot b_{i,j}$ and is a continuous, concave and strictly increasing function of parental investment $b_{i,j}$ that maps parental investment into child income; the function is parameterised by the income level of the parent.

If child status was exogenous (independent of child income), we could guarantee an interior solution to this maximisation problem by imposing the following two restrictions on the curvature of the functions involved. Writing $\sigma_{x,y}$ as the elasticity of x with respect to y we need:

$$\sigma_{u,f} \cdot \sigma_{f,b_{i,j}} < \sigma_{a,n_i} \quad (2.5)$$

$$- (1 + \sigma_{a,n_i}) n_i (b_i + \beta) u''(c_{i,p}) - (\sigma_{a,n_i})^2 u'(c_{i,p}) > 0 \quad (2.6)$$

Where $c_{i,p} = z_i - \sum_{j=1}^{n_i} (b_{i,j} + \beta)$ is the consumption level of the parent. In words, the inequalities say that the elasticity of the well-being of children with respect to parental investment must be lower than the dilutive effect of additional children (otherwise the agent would let $n_{i,j}$ go to 0 and $b_{i,j}$ to infinity, the opposite does

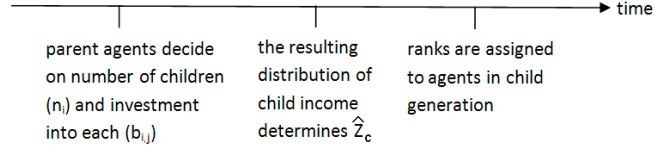


Figure 2.6: *Time sequence of the modelled process.*

not happen because of the presence of the fixed cost of children $\beta > 0$), and that $u(\cdot)$ must be sufficiently concave (otherwise the parent agent would not demand any children at all because $\alpha < 1$). Inequalities 2.5 and 2.6 shall be assumed to hold throughout. We shall also impose $0 > \sigma_{a,n_i} > 1$. For the derivation see Chapter 1.

The timing of the model is the following: first, parent agents simultaneously choose the (continuous) number of children to have and the levels of parental investment for each. These investments then determine the income levels of the children and thus the income distribution in the child generation. Second, the support of the income distribution in the child generation becomes the set of income thresholds of the status function of the child generation. Thirdly, children enter the lottery for status based on their own income level and the income thresholds of the child generation.

This timing is illustrated in Figure 2.6.

The setting I have chosen can perhaps be described as a ‘meritocracy with a class system’, the challenge to the parent being which class to target with each child; importantly I assume that there is no competition for status among members of a class. In contrast to Chapter 1 the focus is not on the competition between these classes, which is an emergent feature of simultaneous decisions of many agents, but on the driving forces of the individual parent’s choice who takes the form of the class system as given.

I should note what the model does not address: I abstract from any kind of inherited status, i.e. agents from different family background but equal income can expect to occupy the same rank in society; there is no mortality risk associated with children; and I also abstract from any other uses of children (apart from altruism) which enter parents’ decision-making process.

2.5 Equilibrium Characterisation

As noted, I focus on pure strategy symmetric Nash equilibria.

Notice first that in this two generational setup only agents of the parent generation have a decision to make. In deciding on how many children to have and how much to invest into each, parent agents determine the support of the child income distribution which via the income thresholds \hat{Z}_c shapes the child status function $S_c(\cdot)$. Notice second that parent agents base their decision on investment levels on their expectation of the form of $S_c(\cdot)$.

In order to be in equilibrium we must therefore have that the child income distribution which the parent agents expect to materialise indeed comes about through the optimal response of the parent agents to this expectation.

The equilibrium interaction of agents is kept simple by the discrete nature of the setup: no features other than the support of the distribution of child incomes counts in the expectations of the parent agents (to be precise: in the determination of $E[S_c(\cdot)]$)⁴.

Note now that in equilibrium there cannot be more than $|R|$ income groups in the child generation. If there were, then the ones with the lowest income levels would be awarded a rank of 0 and given the assumed utility function this means the well-being of children in these income groups would also equal 0. Foreseeing this, no parent agent would invest an amount in any of his children that lead to a child income level associated with one of these 0-rank income groups. Formally:

Lemma 5 MAXIMUM NUMBER OF INCOME GROUPS IN THE CHILD GENERATION

In any equilibrium there are at most $|R|$ income groups in the child generation.

Proof: By contradiction. Suppose there were more. Then by shifting expenditure from children in the income group with the lowest income level to parent consumption, some agents can increase their utility and hence the putative equilibrium is destroyed. \square

⁴The interplay would be more complicated for instance if the value of a rank depended on how many agents share it or how many have a lower rank etc.

In order to have that the optimal choices of the agents given the child income schedule \bar{Z} bring about this schedule we need two conditions to hold:

- (a) that no agent has an incentive to invest in one or more of his children an amount that leads to a child income level not in the schedule \bar{Z} (as agents are homogenous within income groups this would mean that otherwise income groups in the child generation would form that are not part of the child income schedule \bar{Z}); and
- (b) that for each child type in the schedule there is at least one income group of parent agents whose members demand it (otherwise this income group does not materialise).

In order to formalise these two conditions that must be met by the child income schedule \bar{Z} , we need to fix the notation of the structure that such a schedule imposes on the decision-making problem of the parent agents. Let the elements of \bar{Z} be \bar{Z}_m where $m \in \{1, 2, \dots, \bar{m}\}$ with $\bar{m} \leq |R|$ and where the ordering of elements is such that $\bar{Z}_m > \bar{Z}_{m+1}$. For a parent agent with income z_i who considers targeting one or more of these child income levels for his children the corresponding investment choices can be written⁵ as $B_{i,m} \equiv f^{-1}(\bar{Z}_m; z_i)$ with an associated expected rank of $E[S_c(f(B_{i,m}; z_i))] = \tilde{r}_m$. For any parental income level z_i we can thus construct an equilibrium ‘child investment schedule’ B_i whose elements are $B_{i,m}$. As to investment choices out of equilibrium, the expected status function $E[S_c(\cdot)]$ becomes $E[S_c(f(b; z_i) \mid f(B_{i,m-1}; z_i) > f(b; z_i) \geq f(B_{i,m}; z_i))] = \tilde{r}_m$.

In order to test whether \bar{Z} forms an equilibrium, let parent agents of each income level $z_{p,i} \in Z_p$ solve the following maximisation problem:

$$\begin{aligned} \max_{n_{i,m}, b_{i,m}} & \left\{ u \left(z_{p,i} - \sum_{m=1}^{|B_i|} n_{i,m} (b_{i,m} + \beta) \right) r_p + \right. \\ & \left. \alpha \cdot a \left(\sum_{m=1}^{|B|} n_{i,m} \right) \left[\sum_{m=1}^{|B_i|} n_{i,m} \cdot u(f(b_{i,m}; z_{p,i})) \tilde{r}_m \right] \right\} \end{aligned} \quad (2.7)$$

subject to the budget constraint $z_{p,i} - \sum_{m=1}^{|B_i|} n_{i,m} (b_{i,m} + \beta) \geq 0$, and to fertility

⁵This is possible because child income is a strictly increasing function of parental investment.

being non-negative: $n_{i,m} \geq 0$. We further impose $b_{i,m} \geq B_{i,m}$ which ensures that \tilde{r}_m is the right value for $E[S_c(b_{i,m})]$.

Let the resulting fertility choice be denoted by $N_i = \{N_{i,1}, \dots, N_{i,m}, \dots, N_{i,|\bar{Z}|}\}$.

In equilibrium, the agent must have no incentive to set the parental investment into any of his children at a level different from the values in his child investment schedule B_i - this is condition (a) above. As we have ruled out investment levels below $B_{i,|\bar{Z}|}$ we can rephrase this sentence as: the agent must have no incentive to increase parental investment into a child with expected rank \tilde{r}_m above $B_{i,m}$ for all m . In other words, either the constraint $b_{i,m} \geq B_{i,m}$ is binding or $N_{i,m} = 0$.

Next, note that the agent will only have an incentive to increase $b_{i,m}$ over $B_{i,m}$ if the derivative of the objective function in 2.7, which I shall denote U_i , with respect to $b_{i,m}$ is positive at $B_{i,m}$. In equilibrium we must therefore have for each z_i and $B_{i,m}$:

$$\frac{\partial U_i}{\partial b_{i,m}} \Big|_{b_{i,m}=B_{i,m}} = -u'(\cdot)r_p + \alpha a(\cdot)u'(\cdot)f'(\cdot)\tilde{r}_m \leq 0 \quad (2.8)$$

Given the concavity of $u(\cdot)$ and $f(\cdot)$, $\frac{\partial U_i}{\partial b_{i,m}} \Big|_{b_{i,m}=B_{i,m}}$ is monotonically decreasing in the child investment level $B_{i,m}$ which in turn is increasing in the income level of this child type \bar{Z}_m . It follows therefore that there exists a threshold value $\underline{\bar{Z}}_{i,m}$ such that Inequality 2.8 is satisfied iff $\bar{Z}_m \geq \underline{\bar{Z}}_{i,m}$. In words, if the income level of child type m is high enough, agents with income $z_{p,i}$ will have no incentive to choose to have children whose income level is higher than that of type m children and lower than that of type $m-1$ children. If this holds for all child types and for parent agents of all income levels then condition (a) is met.

We can define $\underline{\bar{Z}}_m = \max_i \{\underline{\bar{Z}}_{i,m}\}$ and formalise this as follows:

Lemma 6 MINIMUM INCOME LEVEL FOR EACH CHILD TYPE

Suppose an equilibrium $\langle Z_p, R, \gamma, \bar{Z} \rangle$ exists; then we have that $\bar{Z}_m \geq \underline{\bar{Z}}_m$ for all m .

Proof: By contradiction. Suppose $\bar{Z}_m < \underline{\bar{Z}}_m$ for some m ; then by the definition of $\underline{\bar{Z}}$ agents of some income group(s) will have an incentive to deviate to some b that is $B_{i,m-1} > b > B_{i,m}$ and the putative equilibrium is destroyed as the actual support of the income distribution of children will differ from the one on which

parent agents based their decisions. \square

The idea behind this equilibrium condition is that by forcing the parental investment level of a child type to at least the level of the highest efficient investment level, there is no agent in the population that would want to deviate to a higher level of investment for that child type. If this holds for all types then only income levels that are noted in \bar{Z} are going to be observed in the child generation. Or, to put it differently, for each child type m parent agents are kept from deviating to a different investment level by, for deviations to lower levels, the loss of rank $\tilde{r}_{m+1} - \tilde{r}_m$ for that child and, for deviations to higher levels, by the fact that $B_{i,m}$ is already inefficiently high.

Next I show that equilibrium condition (b), which states that all child types in \bar{Z} must be observed in equilibrium, requires that child income levels are not too close. The logic is the following: as has just been established in Lemma 6, parental investment levels for each child type are weakly above their privately efficient level; equivalently we can say that at $B_{i,m}$ the utility of any parent agent who chooses to have children of type m (that is we have $N_{i,m} > 0$) is decreasing in the income level of child type m . Leaving everything else the same, the utility of these agents would therefore decrease if one was to raise \bar{Z}_m . The important incentive to notice here is that as one tests higher and higher levels of \bar{Z}_m for compatibility with the equilibrium, at some point these agents will switch from type m to type $m-1$ (and that implies from $N_{i,m} > 0$ to $N_{i,m} = 0$), as the difference in parental investment level $B_{i,m-1} - B_{i,m}$ becomes ever smaller (that is the difference in cost of the two child types vanishes) while the status gain $\tilde{r}_{m-1} - \tilde{r}_m$ remains unchanged (which implies that the relative return on investments into type $m - 1$ children increases).

We can therefore define another threshold value, called $\bar{\bar{Z}}_m$, such that for $\bar{Z}_m > \bar{\bar{Z}}_m$ we have $\sum_i N_{i,m} = 0$. In words, if the income level of child type m is too close to that of the next higher child type then its lower cost does not compensate for the lower expected rank it carries and thus no parent agent demands this child type. Note that we have $\bar{\bar{Z}}_m < \bar{Z}_{m-1}$.

Supposing the condition in Lemma 6 is satisfied, existence of an equilibrium therefore also needs the following to hold:

Lemma 7 MAXIMUM INCOME LEVEL FOR EACH CHILD TYPE

Suppose an equilibrium $\langle Z_p, R, \gamma, \bar{Z} \rangle$ exists; then we have that $\bar{Z}_m \leq \bar{\bar{Z}}_m$ for all m .

Proof: By contradiction. Suppose $\bar{Z}_m > \bar{\bar{Z}}_m$ for some m ; then by the definition of $\bar{\bar{Z}}_m$ no agents will demand children of type m and so the support of the child income distribution would not be equal to \bar{Z} and hence the actual $E[S_c(\cdot)]$ will differ from the $E[S_c(\cdot)]$ underlying the maximisation problem in 2.7. With expectations thus rendered inconsistent the putative equilibrium is destroyed. \square

Combining these two Lemmata we arrive at a formalisation of the equilibrium notion.

Proposition 5 CHARACTERISTICS OF A PURE STRATEGY SYMMETRIC NASH EQUILIBRIUM

\bar{Z} constitutes a Nash equilibrium in pure strategies iff, given \bar{Z} , we have for each m both $[\bar{Z}_m, \bar{\bar{Z}}_m]$ nonempty and $\bar{Z}_m \in [\bar{Z}_m, \bar{\bar{Z}}_m]$. We have $\bar{m} \leq |R|$.

Proof: If \bar{Z} constitutes a Nash equilibrium then by Lemmata 6 and 7 we must have for each m both $[\bar{Z}_m, \bar{\bar{Z}}_m]$ nonempty and $\bar{Z}_m \in [\bar{Z}_m, \bar{\bar{Z}}_m]$. Further, from $Z_c = \bar{Z}$ follows $\bar{m} = |Z_c|$ and by Lemma 5 we know that $|Z_c| \leq |R|$.

Conversely, if the conditions in said Lemmata are met, then we have both that $\sum_i N_{i,m} > 0 \forall m$ and that all observed child income levels are elements of \bar{Z} . Given \bar{Z} , therefore the effects of the choices of the agents are consistent with their expectations; further, no agent has an incentive to deviate from this choice and hence we have a Nash equilibrium. \square

By providing the parent agents with a menu of child types $\langle B_{i,m}, E[S(B_{i,m})] = \tilde{r}_m \rangle$ or, in shortened form, $\langle B_i, \tilde{R} \rangle$, the equilibrium effectively socialises half of the decision problem of the parent agent: they need no longer decide on the parental investment levels, but only on how many children to have of each child type m .

We can now move on to the analysis of how the individual parent agent makes his decision in such an equilibrium.

2.6 Individual Decision-Making in Equilibrium

It is helpful to note first what agents would like to do - but cannot do - in equilibrium: they would all like to have children of the highest rank and invest the privately efficient amount. This being below the required amount save perhaps for the very rich, they are left with the choice of either accepting the high price of the highest rank, i.e. child type $\langle B_{i,1}, \tilde{r}_1 \rangle$, and cutting back on the number of children to have (i.e. a low $N_{i,1}$ and $N_{i,m} = 0$ for $m > 1$) or relinquishing the dream of top ranked children and moving to cheaper child types or a mix of expensive and cheap child types.

Case 1: Type 1 children have highest Return Ratio

When will an agent choose to have only children of the highest child type? The answer lies, as in the example at the beginning, in the ‘return ratio’ of a child type which I define as child expected utility level over cost to parent, $\frac{u(f(B_m, z)) \tilde{r}_m}{B_m + \beta}$.

A sufficient condition for an agent with income z_i to only demand children of type 1, i.e. $\langle B_{i,1}, \tilde{r}_1 \rangle$, is satisfied if the return ratio of the type 1 children is the highest of all types:

$$N_{i,m} = 0 \quad \forall \quad m > 1 \quad \text{if} \quad \frac{u(f(B_{i,1}; z_i)) \tilde{r}_1}{B_{i,1} + \beta} > \frac{u(f(B_{i,m}; z_i)) \tilde{r}_m}{B_{i,m} + \beta} \quad \forall \quad m > 1 \quad (2.9)$$

To prove this suppose otherwise, i.e. the agent chooses a positive number for some $N_{i,m}$ for an $m > 1$. But then, fixing his total expenditure on children, the agent can move resources to children of type 1 by the following formula: $\Delta N_{i,m}(B_{i,m} + \beta) + \Delta N_1(B_{i,1} + \beta) = 0$, which implies $\frac{\Delta N_{i,m}}{\Delta N_1} = -\frac{B_{i,1} + \beta}{B_{i,m} + \beta}$. A shift of resources will thus lower the total number of children ($B_{i,1} > B_{i,m}$), and therefore $a\left(\sum_{m=1}^{|B_i|} N_{i,m}\right)$ will increase. The change in $\left[\sum_{m=1}^{|B_i|} N_{i,m} \cdot u(f(B_{i,m}; z_i)) \tilde{r}_m\right]$ can be written as $\Delta N_1 \cdot u(f(B_{i,1}; z_i)) \tilde{r}_1 + \Delta N_{i,m} \cdot u(f(B_{i,m}; z_i)) \tilde{r}_m$. To check whether this is positive for $\Delta N_1 > 0$ use the equation derived before to arrive at $u(f(B_{i,1}; z_i)) \tilde{r}_1 - \frac{B_{i,1} + \beta}{B_{i,m} + \beta} u(f(B_{i,m}; z_i)) \tilde{r}_m \geq 0$ which can be rewritten as $\frac{u(f(B_{i,1}; z_i)) \tilde{r}_1}{B_{i,1} + \beta} \geq \frac{u(f(B_{i,m}; z_i)) \tilde{r}_m}{B_{i,m} + \beta}$ and which by assumption has ‘>’ instead of ‘ \geq ’. Therefore a deviation is profitable and we cannot have $N_{i,m} > 0$ for $m > 1$.

This last paragraph is the formal equivalent of the claim made in the discussion of the example: if both return ratio effect and dilution effect pull into the same direction as is the case if the child type with the highest income level offers the highest return ratio, then the agent will only demand this child type.

Case 2: Type 2 children have highest Return Ratio

As introduced in the example, agents take the child income schedule \bar{Z} and the corresponding parental investment level schedule B_i as given and balance high return ratios and dilution of their altruistic feelings by the number of their children.

We can now analyse with greater rigour the situation in which return ratio and dilution must be traded-off. Putting the assumption of Equation (2.9) aside, imagine now that, as in the example, the expected rank values \tilde{R} as well as the curvatures of $u(\cdot)$ and $f(\cdot; z_{p,i})$ are such that the return ratio of the child type with the second highest income level (type 2) is the highest of all child types. Notice first that, for parent agent $z_{p,i}$ at least, any child type $m > 2$ is dominated by type 2; the same logic as in Equation (2.9) applies. We thus need only consider types 1 and 2.

We can gain an understanding of the agent's choice by considering a deviation from the putative choice of having only type 1 children. Let the optimal number of type one children given that only type 1 children can be demanded be denoted by $\hat{N}_{i,1}$. Then let the agent look for the optimal deviation from this putative choice under the constraint that total investment in children is fixed⁶. I let ΔN_2 denote the number of type 2 children the agent would opt to have if he were allowed to deviate from this putative choice.

$$\max_{\Delta N_2} \left\{ \alpha \cdot a \left[\hat{N}_{i,1} + \left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \Delta N_2 \right] \left[\left(\hat{N}_{i,1} - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \Delta N_2 \right) u(f(B_{i,1}, z)) \tilde{r}_1 + \Delta N_2 u(f(B_{i,2}, z)) \tilde{r}_2 \right] \right\} \quad (2.10)$$

$$\text{subject to } 0 \leq \Delta N_2 \leq \frac{B_{i,1} + \beta}{B_{i,2} + \beta} \hat{N}_{i,1}.$$

⁶This constraint simplifies the exposition by keeping parent consumption constant and does not imply a qualitative change to the decision problem.

We have the following expression for the first derivative which I shall call Q :

$$Q \equiv \alpha \cdot a' \left[\hat{N}_{i,1} + \left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \Delta N_2 \right] \left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \\ \left[\left(\hat{N}_{i,1} - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \Delta N_2 \right) u(f(B_{i,1}, z)) \tilde{r}_1 + \Delta N_2 u(f(B_{i,2}, z)) \tilde{r}_2 \right] + \\ \alpha \cdot a \left[\hat{N}_{i,1} + \left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \Delta N_2 \right] \left[u(f(B_{i,2}, z)) \tilde{r}_2 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} u(f(B_{i,1}, z)) \tilde{r}_1 \right]$$

Abbreviating $\left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \left[\left(\hat{N}_{i,1} - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \Delta N_2 \right) u(f(B_{i,1}, z)) \tilde{r}_1 + \Delta N_2 u(f(B_{i,2}, z)) \tilde{r}_2 \right]$ to A and $u(f(B_{i,2}, z)) \tilde{r}_2 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} u(f(B_{i,1}, z)) \tilde{r}_1$ to B as well as writing N for the total number of children $\hat{N}_{i,1} + \left(1 - \frac{B_{i,2} + \beta}{B_{i,1} + \beta} \right) \Delta N_2$, we can simplify this expression:

$$Q = \alpha \cdot a'(N)A + \alpha a(N)B \quad (2.11)$$

As the agent increases ΔN_2 the weight of type 2 children in his portfolio of children increases and as a consequence (I) his total number of children increases (because we keep his total investment into children fixed) leading to more serious dilution and (II) the average return ratio of his investment increases. The first derivative Q captures these two effects in the negative first summand and positive second summand, respectively.

The first derivative is negative if the dilution effect outweighs the return effect; this implies that type 1 children are preferable to type 2 children and so at the margin the agent would want to reduce ΔN_2 . Rearranging Equation (2.11) we can see that this is the case iff the elasticity of $a(\cdot)$ with respect to its argument is greater than a certain threshold $\hat{\sigma}_{a,N}$:

$$\sigma_{a,N} \equiv \frac{-a'(N)N}{a(N)} > \frac{BN}{A} \equiv \hat{\sigma}_{a,N} \quad \Leftrightarrow \quad Q < 0 \quad (2.12)$$

It is straightforward to show that $\hat{\sigma}_{a,N}$ is increasing in the difference of the return ratios and in ΔN_2 and that it tends to unity as ΔN_2 goes to infinity. The intuition is that the more of the cheap child type the agent has, the less important dilution becomes as dilution hurts more with expensive (high utility) children. As his portfolio of child types changes, dilution thus hurts ever less at the margin.

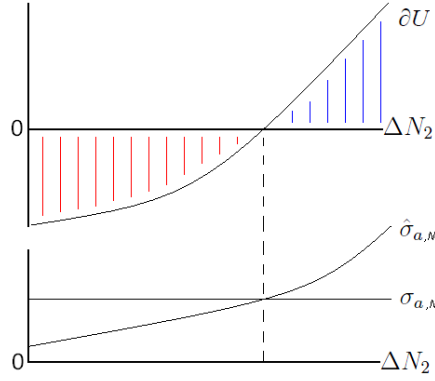


Figure 2.7: The two graphs show the effect of a deviation from demanding only type 1 children (measured by ΔN_2) when type 2 children have a higher return ratio. The lower graph plots $\sigma_{a,N}$ and $\hat{\sigma}_{a,N}$ for the case of $a(n) = n^{-\epsilon}$ with $\epsilon \in [0, 1)$ and the upper graph shows the marginal change in utility. In this example the dilution effect dominates the return ratio effect for small deviations and is dominated for large deviations.

To illustrate, consider the case where $a(n) = n^{-\epsilon}$ with $\epsilon \in [0, 1)$. Since $\hat{\sigma}_{a,N}$ is increasing in ΔN_2 while $\sigma_{a,N} = \epsilon$ is a constant it is clear that either the first derivative is always positive/negative or it is negative for low levels of ΔN_2 and positive for high ones. With $a(n) = n^{-\epsilon}$ we can thus only have corner solutions in the decision problem of Equation (2.10). In words, given $a(n) = n^{-\epsilon}$ with $\epsilon \in [0, 1)$ the agent would either choose $\Delta N_2 = 0$ or $\Delta N_2 = \frac{B_{i,1}+\beta}{B_{i,2}+\beta} \hat{N}_{i,1}$ at which latter point the agent would not demand type 1 children. This is represented graphically Figure 2.7.

Moving on to $a(n) \neq n^{-\epsilon}$, there are forms of $a(n)$ such that interior solutions may arise. Interior solutions lead the agent to demand positive numbers of two child types and therefore mean differential treatment of children.

Technically, the necessary condition is that plotting $\sigma_{a,N}$ on ΔN_2 , $\sigma_{a,N}$ has to cross $\hat{\sigma}_{a,N}$ at least once from below, say at the points in $\{\Delta \check{N}_2\}$, while the sufficient condition is that $\max_{\{\Delta \check{N}_2\}} [\int_0^{\Delta \check{N}_2} \partial U(\Delta N_2) d\Delta N_2] > \max(0, \int_0^{\Delta \bar{N}_2} \partial U(\Delta N_2) d\Delta N_2)$ where I write $\Delta \bar{N}_2 = \frac{B_{i,1}+\beta}{B_{i,2}+\beta} \hat{N}_{i,1}$ for the highest possible deviation with fixed expenditure on children. A possible case is depicted in Figure 2.8.

Note that the solution one arrives at in this way need not be the final choice of the agent as we are still keeping total investment into children fixed at the level optimal given $\hat{N}_{i,1}$.

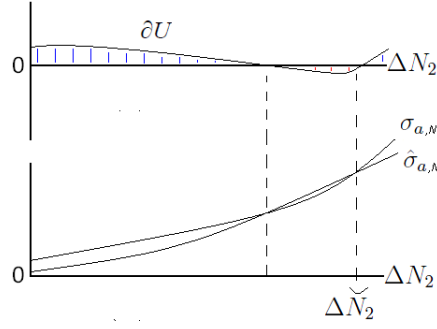


Figure 2.8: These diagrams repeat the exercise of Figure 2.7 with an example of $a(n)$ that may allow for differential treatment of children.

Case 3: Type $m > 2$ children have highest Return Ratio

To generalise further, consider now the case where there is more than one type with higher parental investment level than the type with the highest return ratio. Here, we can first employ the following algorithm to narrow the set of candidates: let the type with the highest return ratio be the first candidate; then take the set of child types with parental investment above that of the candidate, pick the type with the highest return ratio on this set and let it be the second candidate; repeat with the set above the second candidate until reaching type 1 (which is always a candidate). This way we discard all types that are dominated by candidates using the logic outlined in the example that saw only type 1 children in demand by the agent (compare Equation (2.9)).

In the case of $a(n) = n^{-\epsilon}$ a second algorithm can be applied, which takes a randomly drawn pair from this set of candidates and checks the choice of the agent if only these two candidates could be chosen. This choice has been shown above to have a corner solution and so we discard the disfavoured candidate and repeat the process until just one candidate survives. We thus arrive at the following proposition:

Proposition 6 SIMPLE FORMS OF ALTRUISM DO NOT ALLOW FOR DIFFERENTIAL TREATMENT OF CHILDREN

Suppose $a(n)$ takes the form of $n^{-\epsilon}$ with $\epsilon \in [0, 1)$, then in any pure strategy symmetric Nash equilibrium parent agents each choose to demand exactly one child type, so that for each agent of income z_i there is some q such that $N_{i,q} = \hat{N}_{i,q}$ and $N_{i,m} = 0$ for $m \neq q$.

Proof: Suppose this was not true. Then take the agent with differential treatment, fix his total expenditure on children and you will find a profitable deviation involving moving resources from positively demanded type(s) to the one type that survives the two algorithms outlined just above this proposition. \square

In this more general case where type m offers the highest return ratio, $a(n) \neq n^{-\epsilon}$ may produce differential treatment of children in the same way as above. Instead of repeating the above analysis, I show an equation showing the intuition of differential treatment in this generalised form. Taking the first order conditions of the agent's maximisation problem with respect to the number of children of any two child types m and n which he demands in equilibrium (that is $N_{i,m} > 0$ and $N_{i,n} > 0$), setting them equal and rearranging, we get an equation characterising the trade-off between two positively demanded child types m and n :

$$a'[N] \left[\sum_q N_{i,q} u(f(B_{i,q}; z_i)) \tilde{r}_q \right] [(B_{i,m} + \beta)^{-1} - (B_{i,n} + \beta)^{-1}] \\ + a[N] \left[\frac{u(f(B_{i,m}; z_i)) \tilde{r}_m}{B_{i,m} + \beta} - \frac{u(f(B_{i,n}; z_i)) \tilde{r}_n}{B_{i,n} + \beta} \right] = 0 \quad (2.13)$$

Since $B_{i,m}$ and $B_{i,n}$ must be final candidates of the algorithm outlined above Proposition 6, we have $B_{i,m} > B_{i,n}$ and $\frac{u(f(B_{i,m}; z_i)) \tilde{r}_m}{B_{i,m} + \beta} < \frac{u(f(B_{i,n}; z_i)) \tilde{r}_n}{B_{i,n} + \beta}$ or the other way round. Thus we see that always one summand is positive and the other negative, and that the equation represents the balancing of investment return and dilution cost.

Some intuition of differential treatment can be captured by contemplating an agent who chooses to have children of two child types and whose dilution function $a(n)$ is illustrated in Figure 2.8. His reasoning must be as follows: The cheaper child type offers a higher return but dilutes altruism more. His dilution function has a low elasticity for small total numbers of children, so he finds it worthwhile to demand some cheap, high-return and dilutive (because they are more for a given expenditure) children. As he, in his mind, increases the number of children the elasticity of his dilution function increases and he sees dilution as an ever more pressing problem; so he then switches to some of the expensive child type.

The Effect of Parent Income

Another question to ask is how agents differ in their evaluation of child types. The only heterogeneity of parent agents being in parental income this is a question about the role of parent income. The important thing to note is that as long as the function that maps parental investment into child income is independent of the income level of the parent agent, that is $f(b; z) = f(b)$, we have that the return ratio of a child type is also independent of parent income; therefore all agents value child types the same and differences in behaviour arise only due to the dilution effect playing off differently at different values of total expenditure on children⁷.

The issue is more interesting if $f(b; z) \neq f(b)$. In particular, suppose that as in Chapter 1 higher parental income increases the elasticity of f with respect to b . This makes $u(f(b; z))$ less concave in b for high income agents than for low income agents and thus changes the return ratios of the child types for agents of different income. Specifically, high parental investment child types will be more attractive to high income parents.

To illustrate, consider the following example involving two child types. Imagine that child type 2 has the highest return ratio from the perspective of a parent agent with low income z_l , i.e. $\frac{u(f(B_{l,2}, z_l))\tilde{r}_2}{B_{l,2} + \beta} > \frac{u(f(B_{l,1}, z_l))\tilde{r}_1}{B_{l,1} + \beta}$. Now we want to twist $f(b, z)$ such that for a rich agent with high income z_h the opposite is true. We thus want the following to hold:

$$\frac{B_{l,1} + \beta}{B_{l,2} + \beta} > \frac{u(f(B_{l,1}, z_l))\tilde{r}_1}{u(f(B_{l,2}, z_l))\tilde{r}_2} = \frac{u(f(B_{h,1}, z_h))\tilde{r}_1}{u(f(B_{h,2}, z_h))\tilde{r}_2} > \frac{B_{h,1} + \beta}{B_{h,2} + \beta} \quad (2.14)$$

This implies the following necessary condition where $\sigma_{x,y}^w$ denotes the elasticity of x with respect to y for an agent of income level z_w :

$$\begin{aligned} & \frac{f^{-1}(\bar{Z}_2; z_l)(1 + \sigma_{f^{-1}, b}^l \frac{\bar{Z}_1 - \bar{Z}_2}{\bar{Z}_2}) + \beta}{f^{-1}(\bar{Z}_2; z_l) + \beta} \approx \frac{f^{-1}(\bar{Z}_1; z_l) + \beta}{f^{-1}(\bar{Z}_2; z_l) + \beta} \\ & > \frac{f^{-1}(\bar{Z}_1; z_h) + \beta}{f^{-1}(\bar{Z}_2; z_h) + \beta} \approx \frac{f^{-1}(\bar{Z}_2; z_h)(1 + \sigma_{f^{-1}, b}^h \frac{\bar{Z}_1 - \bar{Z}_2}{\bar{Z}_2}) + \beta}{f^{-1}(\bar{Z}_2; z_h) + \beta} \end{aligned} \quad (2.15)$$

⁷In the simple illustration of the case of $a(n) = n^{-\epsilon}$ in Figure 2.7 for instance, depending on the possible maximum deviation one or the other of the two corner solutions may be chosen by the agent.

Comparing the expression on the left and the one on the right of Inequality 2.15 it follows that the inequality holds if $\sigma_{f-1,b}^l$ is sufficiently larger than $\sigma_{f-1,b}^h$ or, equivalently, $\sigma_{f,b}^h$ is sufficiently larger than $\sigma_{f,b}^l$. If this difference is sufficiently large then not only Inequality 2.15 holds but also Inequality 2.14 as required.

2.7 Uncertainty over Child Rank and Incidence of Differential Treatment

In this section, I shall show that high uncertainty over child rank discourages differential treatment of children and eventually abolishes it. Consider first the extreme case of complete uncertainty.

When the social success of a child is completely uncertain or $\gamma = 1$ (that is when parent agents cannot influence the probability distribution of the rank a child is going to be assigned apart from investing at least $B_{i,|\bar{Z}|}$ so that the child is not assigned a rank of 0), then child rank is effectively exogenous.

In this case parental investment only varies the consumption utility of the child, therefore the utility return of parental investment is strictly concave which implies that no two levels of parental investment will have the same marginal utility and hence the maximisation problem of the agent stated in Equation 2.4 has a single interior solution. Parent agents with different income levels may invest different amounts in their children in equilibrium, but any one parent agent will invest the same in all his children. There is thus no differential treatment of children under complete uncertainty.

Differential treatment of children is absent also when uncertainty over child rank is high yet not complete. We can expect to find a certain threshold level of uncertainty, $\hat{\gamma}$, such that there is no differential treatment of children in equilibrium if uncertainty is at least as high as $\hat{\gamma}$.

The idea behind this claim is that greater uncertainty discourages high levels of parental investment by lowering the expected return - a child of the highest income can only lose when ranks are more random - while it encourages higher parental investment into children of the lower expected ranks - a child of the lowest income can only gain from ranks being more random. As a result parental

investment levels are less differentiated across child types and hence the dilution effect plays a lesser role in the decision-making of parents. If the dilution effect becomes sufficiently unimportant, then parents effectively base their decisions on the return ratio effect only; and in that case a single child type will be chosen by each parent.

If we are more specific about the characteristics of the dilution function $a(n)$, we can be more specific about the threshold uncertainty $\hat{\gamma}$. Imagine for instance that the elasticity of $a(n)$ with respect to total number of children, $\sigma_{a,N}$, is fairly low for low total number of children N but increases fast and is bounded above by $\bar{\sigma}_{a,N} < 1$ (see bottom part of Figure 2.9).

In particular, imagine that differential treatment comes about in the following form. The agent has children of two types, j and $k > j$ (it must therefore be that type k has the highest return ratio and type j , being more expensive, is chosen to avert some dilution). At the hypothetical choice under the constraint of demanding only the more expensive child type (represented by $\hat{N}_j > 0$, $N_m = 0$ for $m \neq j$ - compare the example maximisation problem in Equation 2.10 where we had $j = 1$ and $k = 2$) we have:

$$\hat{\sigma}_{a,N}|_{N=\hat{N}_j} > \sigma_{a,N}|_{N=\hat{N}_j} \quad (2.16)$$

and for any fertility higher than the level associated with this choice, i.e. $N > \hat{N}_j$, we have that $\sigma_{a,N}$ is increasing faster than $\hat{\sigma}_{a,N}$, formally:

$$\frac{\partial \hat{\sigma}_{a,N}}{\partial N}|_{\hat{N}_k \leq N < \hat{N}_j} < \frac{\partial \sigma_{a,N}}{\partial N}|_{\hat{N}_k \leq N < \hat{N}_j} \quad (2.17)$$

Figure 2.9 illustrates a case in which an agent with such a $a(n)$ would want to deviate from the hypothetical choice \hat{N}_j (note that therefore the Figure does not show the final solution, only that the solution will involve differential treatment).

The way that a higher γ will abolish differential treatment is by raising $\hat{\sigma}_{a,N}$. In Proposition 7 below I show that higher uncertainty reduces the distance between $B_{i,j}$ and $B_{i,k}$ and thus reduces the relative dilution disadvantage of the higher return child type $\langle B_{i,k}, \tilde{r}_k \rangle$. This manifests itself in the diagram as a higher $\hat{\sigma}_{a,N}$ (the dilution effect dominates the return ratio effect iff $\sigma_{a,N} > \hat{\sigma}_{a,N}$).

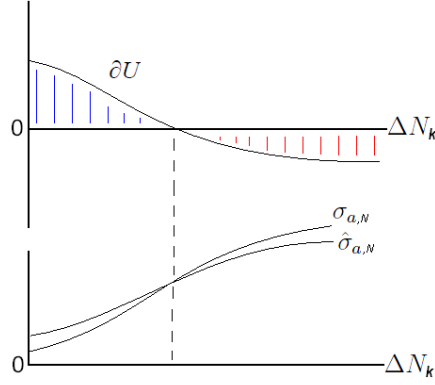


Figure 2.9: *These diagrams show a simple way in which differential treatment can arise; see explanation in the text.*

Suppose for the simplicity of exposition that the equilibrium child income schedule \bar{Z} is equal to the set of lower bounds on child incomes \underline{Z} . We can then derive the following proposition.

Proposition 7 MINIMUM UNCERTAINTY FOR NO DIFFERENTIAL TREATMENT

Suppose $a(n)$ has the properties in Inequalities 2.16 and 2.17 and $\bar{Z} = \underline{Z}$, then $\exists \hat{\gamma}$ such that for $\gamma \geq \hat{\gamma}$ there is no differential treatment in equilibrium.

Proof: From $\tilde{r}_m = (1 - \gamma)r_m + \gamma AM(r)$ where $AM(r)$ is the arithmetic mean of the set of all r_m follows that an increase in γ decreases \tilde{r}_m if $r_m > AM(r)$ and vice versa. As a consequence $\tilde{r}_m - \tilde{r}_{m+1}$ decreases for all m . Further, each element \bar{Z}_m of \bar{Z} is given by $f(b_{i,m}; z_i)$ where $b_{i,m}$ is the solution to the following equation for some z_i (compare Inequality 2.8):

$$-u' \left(z_i - \sum_{m=1}^{|B_i|} n_{i,m}(b_{i,m} + \beta) \right) r_p + \alpha a(\cdot) u'(f(b_{i,m}; z_i)) f'(b_{i,m}; z_i) \tilde{r}_m = 0 \quad (2.18)$$

From this follows that $\bar{Z}_m = f(b_{i,m}; z_i)$ is increasing in \tilde{r}_m . This in turn implies that higher γ lowers $\bar{Z}_m - \bar{Z}_{m+1}$ for all m . Therefore the difference both between \bar{Z}_j and \bar{Z}_k and between $u(\bar{Z}_j)\tilde{r}_j$ and $u(\bar{Z}_k)\tilde{r}_k$ decreases. With the differences between both decreasing, the direction of the return ratios $\frac{u(\bar{Z}_j)\tilde{r}_j}{\bar{Z}_j + \beta}$ is indeterminate. If an increase in γ changes the return ratios such that we move from $\frac{u(\bar{Z}_j)\tilde{r}_j}{\bar{Z}_j + \beta} < \frac{u(\bar{Z}_k)\tilde{r}_k}{\bar{Z}_k + \beta}$ to $\frac{u(\bar{Z}_j)\tilde{r}_j}{\bar{Z}_j + \beta} > \frac{u(\bar{Z}_k)\tilde{r}_k}{\bar{Z}_k + \beta}$ then the return ratio and dilution effect pull into the same direction and there is no scope for differential treatment. If,

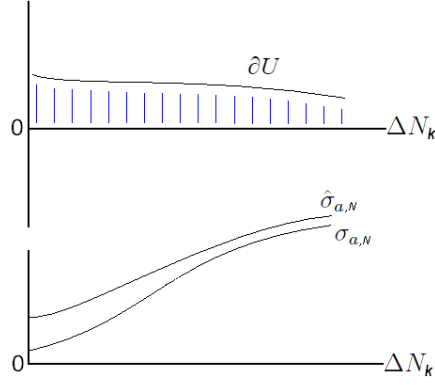


Figure 2.10: *In conjunction with Figure 2.9, these diagrams demonstrate how a higher γ , via raising $\hat{\sigma}_{a,N}$, disposes of differential treatment.*

however, an increase in γ preserves the original ranking of return ratios, then the decrease in the difference between $u(\bar{z}_j)\tilde{r}_j$ and $u(\bar{z}_k)\tilde{r}_k$ implies that

$$\hat{\sigma}_{a,N}|_{N=\hat{N}_j} = \frac{u(\bar{z}_k)\tilde{r}_k - \frac{B_{i,j}+\beta}{B_{i,k}+\beta}u(\bar{z}_j)\tilde{r}_j}{u(\bar{z}_j)\tilde{r}_j - \frac{B_{i,j}+\beta}{B_{i,k}+\beta}u(\bar{z}_j)\tilde{r}_j} \quad (2.19)$$

is increasing in γ . We also know that $\hat{\sigma}_{a,N}$ is increasing in ΔN_k . Since for γ high enough the upper bound on $\hat{\sigma}_{a,N}$ approaches 1 while the upper bound on $\sigma_{a,N}$ is fixed at $\bar{\sigma}_{a,N} < 1$, it follows that, for some $\hat{\gamma}$, $\gamma \geq \hat{\gamma}$ implies that $\hat{\sigma}_{a,N} > \sigma_{a,N}$ for all values of N and for parent agents of any income z_i for all child types j and k . In that case there is no differential treatment of children in equilibrium. \square

If uncertainty is high enough, the agent will deviate from the hypothetical solution $\hat{N}_j > 0$, $N_m = 0$ for $m \neq j$ to having only children of the highest return ratio type: $\hat{N}_k > 0$, $N_m = 0$ for $m \neq k$. The dilution effect which made having some more expensive children with lower return ratio attractive is dominated. This is illustrated in Figure 2.10.

2.8 Discussion

The preceding sections have shown that increasing returns to scale induced by a concern for social status (or relative concerns) can lead to differential treatment

of children if the altruistic feelings of parents towards each child are declining in certain ways as the total number of children of the parent increases. Interestingly, even though the convexity of status returns plays a role by giving the higher type children a better return ratio, differential treatment arises precisely because ranks do not convexify the investment into a single child enough; the return of the high child type is too low to displace the lower type children completely.

The virtual disappearance of differential treatment in modern times can be attributed to technological changes: faster growth in technology as well as a shift of value creation to human capital intensive processes have, from the point of view of the parent, lead to greater talent and idiosyncratic risk associated with parental investment into children. The ensuing greater uncertainty over the future rank of a child is shown in the model to lower the incidence of differential treatment by reducing the cost differences between children and thus weakening the influence of the dilution effect, which stems from the fact that altruism towards any one child decreases with the number of children.

One can generalise the fundamental message of the model: whatever the mechanism behind differential treatment, it is a fair guess to say that it builds on differences between children, be they exogenous or endogenous. Greater uncertainty over the future path of children, which makes children more alike in expectation, lowers these differences and hence removes incentives for differential treatment.

Within the animal kingdom, humans have comparatively few children each, making the abstraction from the integer constraint humans face in their choice of fertility little realistic. In fact, I believe the integer constraint is a fundamental reason for differential treatment of children. While it is not explicitly modelled in the preceding sections, one can easily imagine an extension of the model that incorporates it. The story of the integer restriction would then, I think, be the following: Imagine a high ranking nobleman in a preindustrial society; he would like to have, say, 1.4 children of his rank, but he is not prepared to cut back his own consumption so much as to finance 2 children of this rank. Instead, he has one child (or son) to succeed him in his rank and one or a few barely noble and cheap children. Greater uncertainty of child ranks also discourages this ‘integer induced differential treatment’ by lowering the variance of parental investment

levels and thus reducing the suitability of cheaper child types for substituting fractions of expensive child types.

In a sense, differential treatment in the model and integer induced differential treatment are complementary explanations for differential treatment as they answer two different questions: the first answers why low status children may have high status siblings while the second answers why high status people may have low class siblings.

Further limitations of the model should be addressed. It is asexual both in the sense that there is no matching and mating and also in the sense that there are no genders. While this simplification may seem to be increasingly acceptable in many modern societies, it certainly is problematic in the type of society where primogeniture and differential treatment was or is common practice. To explore the origins and implications of gender roles from this perspective is an avenue for future research.

I should also add that in real life children, especially in societies of the past, may come into existence for reasons other than those captured by the notion of altruism of the model: they may be the byproduct of sexual desire, replacements in waiting in case the chosen heir would die prematurely, labourers on the family farm, providers of retirement income, tokens in marriage arrangements and so on.

Chapter 3

Co-Evolution of Institutions and Preferences: the case of the (human) mating market

3.1 Introduction

Recent years have seen a surge of studies that show how natural selection operating on both genes and institutions (which shall be understood as the rules, habits and other culturally transmitted norms that individuals of a species follow when interacting with each other) can lead to co-evolution of the two (see e.g. Boyd and Richerson [2002], Bowles et al. [2003], Bowles [2006], Choi and Bowles [2007] and Gintis [2007] - for early proponents see for instance Cavalli-Sforza and Feldman [1981], Durham [1992], Feldman and Zhivotovsky [1992] and Soltis et al. [1995]). The general result can be stated as follows: evolution creates institutions that are complementary to the genetic setup of the (sub)population. In particular, the literature has shown how institutions can be complementary to certain genetically fixed behaviours - mainly in the human domain. Consider Bowles et al. [2003] as an example: the authors study the emergence of genetically induced behaviours that are beneficial to the group and costly to the

individual. While reproductive differentials of individuals thus drive these behaviours towards extinction, group level selection works for their proliferation. Institutions such as resource sharing or segmentation reduce the variance of reproductive success within groups and thus weaken the force of selection on the level of individuals; the emergence of these institutions depends on the existence of such group-beneficial traits and these in turn may only be able to proliferate if these institutions are in place.

Note that the very existence of groups can for some species be interpreted as a part of the institutional setup.

This paper, focussing on mating preferences and the mating market, adds to this literature by showing that evolution may also create institutions that are complementary to the mate choice based on genetically fixed preferences, which can be interpreted as genetically coded information processing.

The model is as follows. In a population of a strictly monogamous species, individuals choose who to form a couple for life with, their choice based on the observable characteristics of the opposing sex (the opposing sex' phenotype). The setup is such that, in its optimal form in terms of maximising evolutionary fitness, the desirability ranking of potential mate types depends on the characteristics of the individual forming this ranking.

This is achieved by good and bad expressions of parental characteristics being sub-modular in terms of the evolutionary fitness of joint offspring; which in turn depends on a dominant-recessive allele setup of the relevant characteristics of individuals. Dominant alleles code for the evolutionarily better phenotype. If one is homozygotic in the recessive allele (carries only the latter), then switching from a mating partner who is also homozygotic in the recessive allele to one who carries at least one dominant allele increases the probability of one's child carrying the more advantageous dominant allele by more than if one oneself carries at least one such dominant allele.

The mentioned optimal preferences will thus result in a matching allocation that sees individuals of different strengths and weaknesses form couples. Such an allocation happens to be evolutionarily efficient in the sense that the speed with which favourable genes spread through the population is maximised as the reproductive differential between good and bad expressions of traits is maximised¹.

¹Note that if parental characteristics were modular instead of submodular the efficient allo-

However, one cannot expect genetically fixed mating preferences to be completely conditionable on the characteristics of the carriers. I therefore introduce the constraint that they cannot depend on the phenotypical characteristics of the carrier other than his gender (and thus also not on his genotype other than the bit that determines his gender). This can be interpreted as a constraint on the complexity of mating preferences.

The population is then assumed to adopt a certain mating partner ‘allocation mechanism’ (or, equivalently, an ‘institutional setup’ of its mating market) which structures the choices to be made by the individuals. In particular, I assume that the allocation takes place following the ‘deferred-acceptance procedure’ of Gale and Shapley [1962]². Given this allocation mechanism, the evolution of mating preferences such constrained is studied. I derive the equilibrium mating preferences and their implications for the evolution of the phenotypical traits based on which individuals rank possible mates.

The result is that under certain conditions this ‘deferred-acceptance procedure’ (DA procedure) produces mating preferences which, conditional on this institutional setup being in place, are optimal for the individuals³ - they form couples as if they were guided by the unconstrained optimal desirability ranking mentioned above. And therefore the matching allocation under the DA procedure is evolutionarily efficient.

Nature may thus be thought of as overcoming the genetic constraint on the complexity of mating preferences by structuring the interaction of the individuals of the species in a certain way, that is by letting the species adopt an appropriate institutional setup. Based on the fact that the DA procedure induces an evolutionarily efficient allocation, an argument is then developed that suggests that group level selection will let it prevail over other institutional setups.

The equilibrium preferences in the DA procedure are interesting because the

cation could be reached with desirability rankings that are independent of the characteristics of the individual making the choice.

²The allocation is a two-sided matching problem with nontransferable ordinal utility (compare Legros and Newman [2007]). Further seminal works of the two-sided matching literature include Knuth [1997], Becker [1981] and Roth and Sotomayor [1992].

³To be precise, conditional on the two having the same genotype, an individual maximising these preferences and an individual with full information and the desire to maximise his evolutionary fitness would do equally well in expectation.

preference of the two genders can be shown to be partly opposing and complementary: if one gender prefers a certain trait over another, individuals of the opposite sex will prefer the other trait. An ensuing application to modern marriage markets of humans (which formed the original motivation for this paper) argues that mating preferences formed in our paleolithic past may be responsible for the anecdotal fact that women are quite reluctant to ‘marry down’ the social ladder while for men traits such as beauty are far more important than social status or intelligence. This result is also interesting as it does not hinge on the dynamics specific to polygynous mating.

This paper is also contributing to the vast literature on sexual selection. Originating with Darwin [1871], sexual selection as a distinct force in evolution has probably received most attention in the form of runaway processes à la Fisher which are based on correlations between genes coding for characteristics and preferences (see Fisher [1930] and for a formal model Lande [1981]; for further extensions see e.g. Pomiankowski and Iwasa [1993]) and the handicap principle which is based on signalling arguments (see Zahavi [1975] and his follow-on papers as well as e.g. Grafen [1990]); both these processes are ruled out by assumption in this model, however, as traits are both assumed to be statistically independent and perfectly observable. Assortative mating similar to the partial assortative mating that emerges in the model below has been studied for instance by Karlin and O’Donald [1978] and Wilson and Dugatkin [1997].

Comparative analysis of alternative mating mechanisms is covered for instance in Andersson [1994]; the perhaps closest related work to the present paper in this field is Servedio and Lande [2006] who study the dynamics of population genetics for polygynous species.

The rest of the paper is structured as follows. The next section introduces the setup of the model in more detail and provides more motivation, then examines the evolution of the modelled traits including preferences under the DA procedure before giving some observation concerning the co-evolution of institutions and preferences as well as other traits. This is followed by the application to modern marriage markets and a short discussion.

3.2 The Model

3.2.1 Setup

Introduction

The modelling approach of this paper is the following: it tracks the evolution of genotypes and determines their equilibrium values for each of a set of institutional setups of the mating market; in a second step, it then asks which of these institutional setups would prevail in an evolutionary process based on group selection.

The analysis applies to sexually reproducing species that form monogamous couples, have moderate cognitive capabilities and whose success in rearing young does not systematically depend on culturally determined characteristics such as inherited wealth or status. The model thus has no distinctively human features, I shall, however, refer to the example of a community of human hunter-gatherers in the stone age as a motivating example.

In the following sections I shall expose the genetic setup of individuals and the population and show how genotypes are linked to reproductive success, followed by an overview of the institutional setup of the mating market.

To fix ideas, imagine the life cycle of a member of the hunter-gatherer community to be as follows: When child is born to a couple, it survives into adulthood with a certain probability. This probability depends on the characteristics of his parents, the more successful they are in life the more likely the child is to survive.

When coming of age, the individual enters the mating market and is there matched with a partner of the opposite sex with whom he or she forms a monogamous couple for life. The number of children who survive into adulthood depends positively and exclusively on the success in life of the parent couple; and success in life in turn depends (systematically) only on the genotypes of the partners. In the end the individual dies.

Genotypes and ‘Success in Life’

As mentioned above, the number and probability of survival into adulthood of children depends on parental characteristics such as access to food, material well-being, health, survival into old age etc. These we shall call summarily ‘success in life’. For evolutionary purposes what matters is the connection of genes and ‘success in life’.

Suppose that we can describe genetic heterogeneity of agents along only four dimensions: gender, a characteristic named L , a characteristic named I , and mating preferences.

As to the first three characteristics I assume for the sake of simplicity of exposition that they are binary: gender naturally can be male (m) or female (f) while L , which we may or may not interpret as looks or beauty, can be good (g) or bad (b) and I , which we may or may not interpret as intelligence or cleverness, can be high (h) or low (l). I assume that the phenotype of an individual is perfectly observable to all individuals in the community including the individual him- or herself.

In terms of genes, one may imagine gender to be the result of the genotype having XX or XY chromosomes while for L and I I assume that their phenotypic expression results from simple autosomal dominance setups: let there be two alleles for L named T and t , the first of which causes the bearer to have the more advantageous expression g ; we thus have the following relationship between genotypes and phenotype (as far as L is concerned): $\{TT, Tt, tT\} \Rightarrow g$ and $tt \Rightarrow b$. Similarly for I , I define alleles U and u and $\{UU, Uu, uU\} \Rightarrow h$ and $uu \Rightarrow l$.

Mating preferences or in short ‘preferences’ take the form of an ordering of all possible phenotypes involving the characteristics L and I . This implies that there is no homosexuality and that there are no preferences over the the potential partner’s preferences. I do not specify the underlying genetics.

To simplify, I assume that the alleles of all four characteristics are inherited independently which implies that, within a subgroup established by conditioning on a phenotypic characteristic, the frequencies of the other characteristics are identical in expectation to the frequencies we observe in the total population.

This ensures for example that if $\frac{3}{4}$ of the population has g and half is male then we have that of the subgroup of individuals with h $\frac{3}{8}$ are male with g , $\frac{1}{8}$ are male with b , $\frac{3}{8}$ are female with g and $\frac{1}{8}$ are female with b .

Suppose that the expected number of surviving children can be expressed as a function of the sum of the values for success in life of the two parents: $E(S|sl_m + sl_f)$ with $\frac{\partial E(S|sl_m + sl_f)}{\partial (sl_m + sl_f)} > 0$, where S stands for number of surviving children and sl_m and sl_f for success in life of male parent and female parent, respectively.

Let us further assume that the gender of the parent has no bearing on how beneficial these traits are and that L and I are equally important to the success in life. As a function of the phenotype we can therefore set (where $x \in \{m, f\}$):

$$sl_x(g, h) > sl_x(g, l) = sl_x(b, h) > sl_x(b, l) \quad (3.1)$$

When a man and a woman have a child in this setup, the child's phenotype with respect to L and I is one of the four types (g, h) , (b, h) , (g, l) and (b, l) . The probability of these depends on the phenotypes of the parents. If we write $Pr((L_c, I_c)|(L_m, I_m), (L_f, I_f))$ for the probability of the child having phenotype (L_c, I_c) , with looks L_c and intelligence I_c , given that the father has phenotype (L_m, I_m) and the mother has phenotype (L_f, I_f) , then from Mendelian rules follows that:

$$\begin{aligned} 1 > P_{L1} &= Pr((g, \cdot)|(g, \cdot), (g, \cdot)) \\ &> P_{L2} = Pr((g, \cdot)|(g, \cdot), (b, \cdot)) = Pr((g, \cdot)|(b, \cdot), (g, \cdot)) = \frac{1}{2} \\ &> P_{L3} = Pr((g, \cdot)|(b, \cdot), (b, \cdot)) = 0 \end{aligned} \quad (3.2)$$

from which follows that $P_{L1} - P_{L2} < P_{L2} - P_{L3}$ which means that the gain of additional probability of a good looking child from being paired with an individual with good looks is smaller for an individual who has himself good looks. Equation 3.2 is illustrated in Figure 3.1.

Similar conditions hold for I and I define equivalents of P_{L1} , P_{L2} and P_{L3} as P_{I1} , P_{I2} and P_{I3} . By construction we have $P_{L2} = P_{I2} = \frac{1}{2} \equiv P_2$ and $P_{L3} = P_{I3} = 0 \equiv P_3$ and to simplify the exposition I also assume that $P_{L1} = P_{I1} \equiv P_1$.

The choice of a dominant/regressive mode of inheritance with two alleles for

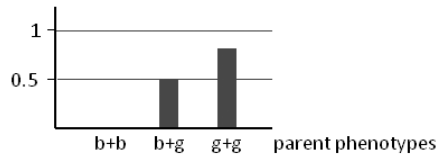


Figure 3.1: *This diagram shows the probability of a child having good, or ‘g’, in L depending on the phenotypes of his parents.*

each trait is a special setup, but the important underlying assumption is that mating with an individual with preferred characteristics improves the expected genetic quality of a child more in the case of agents who lack this preferred characteristic; to economists this would go by the name of diminishing returns or submodularity. And this is true for many modes of inheritance.

Mating Market

Imagine the mating market to be such that when a generation comes of age (say at a certain date every year) men and women are matched in monogamous pairs that stay together for life. To focus the exposition we assume equal numbers of men and women and will work with the expectation of the distribution of types. This means that I essentially work with large populations. That is admittedly a potentially dangerous abstraction in an evolutionary context.

The matching is assumed to take place following the DA procedure, according to which individuals of one gender propose to form a partnership to an individual of the other gender who in turn decides whether to accept or reject the proposal. Let individuals of the former gender be called ‘proposers’ and those of the latter ‘receivers’. Proposing takes place in rounds: in the first round each proposer proposes to his favourite receiver (exactly one proposal per proposer) and after receiving all proposals each receiver rejects all but one proposal if any. The rejected proposers then propose to their second most favourite receiver and once all these proposals are received each receiver again rejects all but one of the new proposals and the kept proposal of the first round. This procedure is repeated until all individuals are matched. By Theorem 1 of Gale and Shapley [1962] this procedure always results in a stable allocation, i.e. one in which no two individuals would want to leave their allocated partner in order to form a

couple among themselves.

In order to ease the language of the following sections, I shall describe the outcome of the DA procedure as if individuals understood its working well enough for proposers not to propose to receivers they know will reject them and for receivers to accept proposals when they know that they cannot get a better one. How they know shall be made clear whenever this argument is invoked.

In the following we shall track the evolution of all four genotypical characteristics (in the form of replicator dynamics) under the DA procedure.

I assume that nature is constrained to condition preferences only on the gender of the individual; in other words, men and women may have different preferences but whether a man or woman is of phenotype g or l in L or of phenotype h or l in I can have no direct effect on his or her preferences (this implies his genotype in terms of u , U , t and T can have no direct effect on the carrier's preferences).

This is of course again a simplification and stands for the underlying assumption that mating preferences are not optimised given complete conditioning on genetic and other characteristics. This I think is a reasonable view.

A note on objectives and information sets: while the individual knows both his own type and that of all other individuals, the genes that determine preferences only know the gender of their carrier, nothing else. For the individual the task is to get matched with an individual of the opposite sex that is as high in the ranking of types as possible, where this ranking is dictated by his preferences.

The task (figuratively speaking) for preferences, however, is to devise the ranking of types that generates the highest possible evolutionary fitness for their carrier (who decides what type to match with based on this ranking). It is therefore in the evolution of preferences that the submodularity of types and the fact that preferences cannot be conditioned on all the characteristics of the carrier matter. These two features of the setup do not matter in any single instance of the matching process.

Benchmark Matching without Complexity Constraint

In order to have a benchmark outcome or scenario with which to contrast the matching outcomes in the following sections, it is worthwhile to consider the outcome if individuals are taken to be rational agents and preferences are not constrained in their complexity. This is the full information case.

Under such circumstances a matching is stable if there is no combination of two agents such that both would want to leave their matched partner in order to form a couple. This implies that preferences will evolve such that the ranking of desired types is equal to the ranking of types based on the expected success in life of the offspring generated by forming a couple with each of these types. The resulting preferences are tabulated below.

Table 1

own phenotype	preferences
(g, h)	$(g, h) \succ (g, l) \sim (b, h) \succ (b, l)$
(g, l)	$(g, h) \succ (b, h) \succ (g, l) \succ (b, l)$
(b, h)	$(g, h) \succ (g, l) \succ (b, h) \succ (b, l)$
(b, l)	$(g, h) \succ (g, l) \sim (b, h) \succ (b, l)$

In terms of matching outcome we have that (g, h) types are only matched with (g, h) types, (g, l) types with (b, h) types (if they are of equal frequency), and (b, l) types with (b, l) types. If (g, l) and (b, h) types are not of equal frequency then the more common type will form couples among itself. The evolutionary implications of such a matching outcome are discussed below.

Note that preferences in this full information case are not conditioned on the gender of the carrier. It only plays a role given the complexity constraint as, within a structured interaction with gender roles, gender may then be associated with and therefore proxy the characteristics that actually matter.

Review of critical Assumptions

In this section I list and explain the set of assumptions that drive the model.

Assumption 1 *The species reproduces in strictly monogamous couples and inherited traits are statistically independent and their phenotypical expression perfectly observable.*

This provides for a simple environment by restricting the number of possible institutional setups for the mating market and ruling out indirect effects of traits. This in turn focuses attention on the interplay of genetic and institutional setups.

Assumption 2 *The fitness ranking of types of mating partners of an individual depends on the genotype of this individual; in particular, type (g, l) individuals have children of higher expected success in life if mating with a type (b, h) rather than a type (g, l) while for type (b, h) individuals this ranking is reversed.*

This is modelled in the form of diminishing returns (compare Equation 3.2 and Figure 3.1): mating with an individual of phenotype g is more valuable to an individual of phenotype b than g .

Assumption 3 *Nature is restricted in the complexity of the genetic setup of mating preferences in that these can only be conditioned on the gender of the carrier of the preferences not his other characteristics.*

The purpose of assumptions 2 and 3 is to let optimal preferences be a non-degenerate function of one's phenotype and, figuratively, to bar nature from giving individuals these optimal preferences. Taken together this implies that there is scope for improvement over genetically fixed mating preferences, a task which, as shall become evident below, may be taken up by institutions.

3.2.2 The Evolution of Gender, L and I

While all four traits (gender, L , I and preferences) evolve simultaneously, it can be shown that some characteristics of the evolution of the first three of these are independent of the institutional setup of the mating market and the evolution of preferences.

As to gender, I assume that initially there are equal numbers of males and females and I show that, as a direct result of Assumption 1, this distribution is stable:

Proposition 8 GENDER RATIO

In equilibrium, genes coding for male and female have the same frequency so that the gender ratio is 1 : 1.

Proof: Note that due to the monogamous setup the distribution of reproductive success is identical for the two genders; together with the fact that gender is inherited independently from other traits this implies that the expected fitness of a male child is equal to that of a female child. The posited ratio thus forms an equilibrium.

Should the expected gender ratio deviate from 1 : 1, children of the gender that forms the longer side of the market will have a lower expected fitness as a portion of its members will not be matched and thus will not reproduce. This lets selection work in its disfavour until the ratio of 1 : 1 is restored. Therefore the posited ratio is the unique equilibrium in the genes coding for gender. \square

The genes governing the traits I and L evolve in a way that lets the population frequencies of g (the ‘good’ version of L) and h (the ‘high’ version of I), denoted P_g and P_h respectively, increase over time. In order to avoid degenerate distributions of I and L , I assume that there is a constant error rate such that when an individual of genotype UU mates, he passes on a copy of u instead of U with a certain (small) probability ϵ . Let the equivalent hold for T and t .

We then have:

Proposition 9 DYNAMICS OF I AND L

P_g and P_h weakly increase over time and are each bounded above by $1 - \epsilon^2 < 1$.

Proof: There is change over time in P_g if at least some individuals of type g are matched to individuals of type g and similarly for b , because only then does the distribution of reproductive success differ across g and b . The worst possible scenario for g is that there are only matches of (g, l) with (g, l) and (b, h) with

(b, h) . In this special case, the success in life of the two types of couples is equal (by Equation 3.1), in all other cases the success in life of the couples having g is higher than that of the couples having b and then P_g increases. The same analysis holds for P_h .

The upper bounds on the prevalence of P_h (P_g) stem from the fact that even a population of individuals of genotype UU (TT) will, in expectation, see a fraction ϵ^2 of their children have genotype uu (tt) and phenotype l (b) due to the above introduced error rate. \square

Note that Proposition 9 makes no statement regarding the average speed with which P_g and P_h approach their upper bounds from their initial value, nor does it imply that the two frequencies will evolve at the same speed. These characteristics of the evolution of I and L depend, as shall be shown below, on the evolution of preferences and institutions.

3.2.3 The Evolution of Preferences

We now turn to how the preferences of both genders evolve. Preferences are set over the four phenotypes (g, h) , (g, l) , (b, h) and (b, l) . It is clear that whatever an individual's own phenotype and genotype, it is beneficial for him or her to prefer a partner with phenotype (g, h) to all other phenotypes. Likewise the individual should prefer all other phenotypes to type (b, l) . Formally we have:

Lemma 8 PREFERENCES OVER TYPES (g, h) AND (b, l)

Preferences will evolve to the following general form:

$$(g, h) \succ X \succ (b, l) \quad X \in \left\{ \left[(g, l) \succ (b, h) \right], \left[(g, l) \sim (b, h) \right], \left[(g, l) \prec (b, h) \right] \right\} \quad (3.3)$$

Proof: Whenever an individual is given the opportunity to choose between being matched with a type (g, h) individual or some other type, he will have higher expected reproductive success if he chooses the type (g, h) individual irrespective of his own genotype; see Equation (3.1). The choice of individuals is based on their preferences and therefore preferences that do not rank (g, h)

highest will result in suboptimal choices and thus eventually be driven out. Likewise preferences that do not rank (b, l) lowest will result in suboptimal choices as expected reproductive success is higher when mating with an individual of some other type. \square

We examine a situation in which the forces of Lemma 8 have already moved preferences to the form of $(g, h) \succ X \succ (b, l)$. We can now deduce the following behaviour.

Proposers of type (g, h) propose to receivers of the same type and get accepted: the receivers accept the best possible match and the proposers foresee this. Since we are working with expectations there are equal numbers of proposers and receivers of this type and so this type enters partnerships among its own members only.

Likewise, proposers of type (b, l) propose to receivers of the same type and get accepted. These individuals realise that type (g, h) proposers and receivers match among themselves and that there are equal numbers of type $(g, l) \cup (b, h)$ proposers and followers. They also realise that preferences are as in Equation 3.3 so that the latter types rather match among themselves than with a (b, l) type individual. So receivers of type (b, l) accept proposers of type (b, l) knowing that they cannot expect to get a better match by waiting and receivers of type (b, l) propose to them because they know that receivers of any other type would reject them.

These results are independent both of the distribution of X in the population and, by the independence of traits in the inheritance process, of the distribution of L and I . We may therefore suppress an exposition of the matching of types (g, h) and (b, l) in what follows.

Since individuals of type (g, h) form couples among themselves as do those of type (b, l) , the individuals that count for the dynamics (or evolution) of X are thus of type (g, l) and (b, h) only. Let a denote the frequency of $X = [(b, h) \succ (g, l)]$ for receivers and b the frequency of $X = [(b, h) \succ (g, l)]$ for proposers. If we find an equilibrium in $\{a, b\}$, then taken together with Equation (3.3) we have characterised the equilibrium distribution of mating preferences.

The important bit to notice is that there is a tension or conflict between preferences that are optimal for a (g, l) type individual and for a (b, h) type individual as both types maximise their evolutionary fitness within the subgroup of type $(g, l) \cup (b, h)$ individuals by their complement type⁴. The solution to this problem is to make use of the information of the opposite sex. This can be made clear with an example for the receivers: if all proposers have certain preferences these will be optimal for one of the two phenotypes, so receivers should accept proposals from this phenotype but not from the other; when all proposers of the phenotype for which the proposer preferences are optimal are matched the remaining proposers and receivers will be of the complement types and thus also form optimal couples.

This idea can be generalised for non-degenerate distributions of preferences. I first review the special case of equal numbers of individuals of the two types (g, l) and (b, h) , i.e. the case of $P_g = P_h$. We can then formalise the solution idea for receivers in the following Lemma:

Lemma 9 DYNAMICS OF RECEIVER PREFERENCES GIVEN $P_g = P_h$

The frequency of receivers with preferences $X = [(b, h) \succ (g, l)]$ denoted a decreases if $b > \frac{1}{2}$, increases if $b < \frac{1}{2}$ and does not change if $b = \frac{1}{2}$ and/or $a = \frac{1}{2}$ where b is the frequency of proposers with preferences $X = [(b, h) \succ (g, l)]$.

Proof: See Appendix.

The proof makes use of the fact that unless $a = \frac{1}{2}$ and/or $b = \frac{1}{2}$ one type of one gender is matched entirely to the preferred partners in $Q = \{(g, l), (b, h)\}$ so that of this gender only individuals of the other type remain. All remaining individuals of the opposite sex are thus forced to match with this type irrespective of their preferences. This is illustrated with an example in Figure 3.2.

The intuition for the dynamics of proposer preferences as opposed to receiver preferences is that they, too, can infer the optimal match of their own phenotype from the preferences of the opposite sex: if all receivers prefer a certain type in Q , then it is opportune for proposers to have preferences that are optimal for this preferred type because if they are accepted the match will be optimal while if they are rejected by the partners they prefer they benefit too by being forced

⁴Compare Assumption 2.

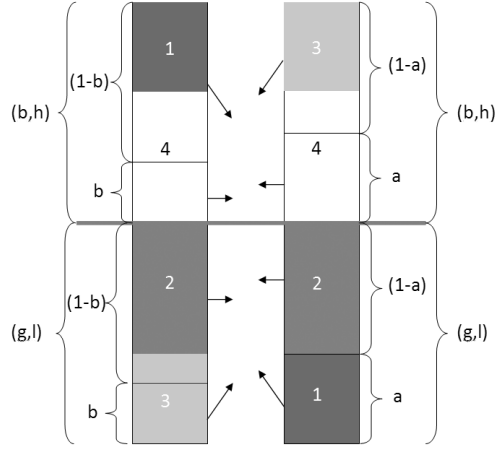


Figure 3.2: This diagram shows how types (g,l) and (b,h) are matched in an example with $a, b < \frac{1}{2}$: the left column represents proposers, the right column receivers. Arrows point in the direction of the preferred type. The numbering shows in which order one can imagine matches to occur.

to mate with the less preferred but optimal complement type in Q^5 .

We arrive at a result similar to the one above:

Lemma 10 DYNAMICS OF PROPOSER PREFERENCES GIVEN $P_g = P_h$

The frequency of proposers with preferences $X = [(b,h) \succ (g,l)]$ denoted b decreases if $a > \frac{1}{2}$, increases if $a < \frac{1}{2}$ and does not change if $a = \frac{1}{2}$ and/or $b = \frac{1}{2}$.

Proof: The proof is very similar to the one in Lemma 9; it checks the same cases and notes the dynamics of b which are the flip side of the dynamics of a . \square

Considering these two results in the replicator dynamics as well as our earlier results we arrive at the following:

Proposition 10 PREFERENCES IN THE DA PROCEDURE GIVEN $P_g = P_h$

There are two stable equilibria in preference frequencies: $\{a=1, b=0\}$ and $\{a=0, b=1\}$. In both of these the mating allocation is as follows: (g,h) proposers with (g,h) receivers, (g,l) proposers with (b,h) receivers, (b,h) proposers with (g,l) receivers, and (b,l) proposers with (b,l) receivers.

⁵Where ‘optimal’ is short for optimal given that (g,h) individuals are not available because they match among themselves.

Proof: From Lemmata 9 and 10 immediately follows that the following frequency pairs are equilibria $\{a=1, b=0\}$, $\{a=0, b=1\}$, $\{a=\frac{1}{2}, b \in [0, 1]\}$ and $\{a \in [0, 1], b=\frac{1}{2}\}$. However, it also follows from the Lemmata that only the first two of these are stable when subjected to repeated small random shocks to the frequencies.

It has been shown above that we always have (g, h) proposers pairing with (g, h) receivers and (b, l) proposers pairing with (b, l) receivers. In the case of $\{a=1, b=0\}$, (b, h) proposers prefer (g, l) partners and (g, l) receivers prefer (b, h) partners so that these two groups form couples. This leaves (g, l) proposers and (b, h) receivers with no choice but to pair. In the case of $\{a=0, b=1\}$, (g, l) proposers and (b, h) receivers willingly form partnerships and (b, h) proposers and (g, l) receivers are forced to mate. In either case the allocation is as shown above. \square

Note that this mating allocation is the one obtained in the benchmark scenario of section 3.2.1 and that it is efficient from an evolutionary point of view in the following sense: the evolution of L is speeded because matches involving two individuals carrying g and two individuals carrying b are (g, h) with (g, h) only and (b, l) with (b, l) only - and this maximises the reproductive differential between the two. The equivalent holds for the evolution of I . The reason is that individuals are matched partly assortatively in the sense that the top and bottom types (in terms of success in life) are paired and that the mediocre types match in cross pairs (g matched with b and h with l). The cross pairing of mediocre types means that they do not count towards the reproductive differential of the expressions of L and I .

When the frequencies of g and h are not equal so that $P_g \neq P_h$, this strong result no longer necessarily holds. The intuition is that whenever an element of X becomes more prevalent then it has to take on a greater relative burden of sub-optimal matches of same types in Q . This is exemplified in Figure 3.3 and formalised in the following proposition.

Proposition 11 POSSIBLE BREAKDOWN OF THE EQUILIBRIUM IF $P_g \neq P_h$
 $\exists r_{upper} > 1, 0 < r_{lower} < 1$ such that if $\frac{P_g}{P_h} > r_{upper}$ or $\frac{P_g}{P_h} < r_{lower}$ then no stable equilibrium of the form in Proposition 10 exists.

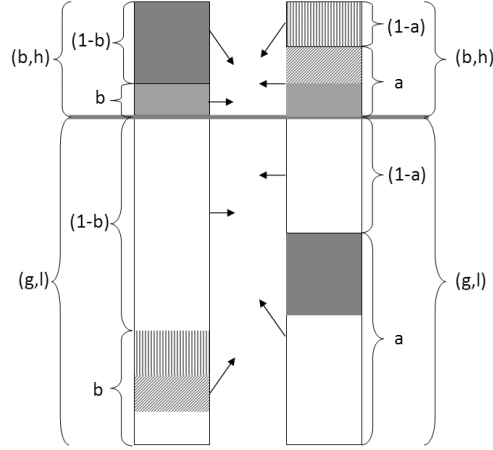


Figure 3.3: *This diagram shows the breakdown of the mechanism that sustains the equilibrium of Proposition 10 when $P_g > P_h$ to a sufficient degree. We have $a > \frac{1}{2}$ and $b < \frac{1}{2}$ so in the case of $P_g = P_h$ we would see $a \rightarrow 1$ and $b \rightarrow 0$; but here the enlarged pool of (g,l) individuals preferring (b,h) individuals over their own type ensures that the absolute number of right matches is the same for both expressions of X which implies that the more frequent expression loses out so that $a \neq 1$.*

Proof: See Appendix.

Note that even when no stable equilibrium exists, the result of Lemma 8 still holds so that while the distribution of preferences does not converge to an equilibrium in terms of X it does converge to the form in Equation 3.3: $(g,h) \succ X \succ (b,l)$. And this implies that, as noted above, we will see (g,h) type individuals matched to (g,h) types only and similarly for (b,l) types. Before discussing the co-evolution of institutions and genetic traits, I shortly review a possible genetic innovation that may be able to restore a stable and efficient equilibrium.

An Extension: Preferences over Preferences

Within the setup of the DA procedure with $P_g \neq P_h$, imagine that by mutation the mating preferences of individuals are no longer a ranking over the phenotypic expression of L and I of the opposite sex only but over the phenotypic expressions of L , I and mating preferences of the opposite sex.

The aim of this section is not to suggest that such preferences will evolve but

to give an example of how genetic innovation may be beneficial in certain institutional setups. Instead of a full analysis therefore, I focus on the implications of a certain set of preferences over preferences. I consider only the dynamics of preferences over (g, h) and (b, l) types.

Denote individuals who prefer individuals of type (g, h) over type (b, l) by \hat{G} and those with opposite preferences by \hat{I} . Then by arguments similar to the ones advanced in Lemma 8 we will arrive at the following general form of preferences:

$$(g, h, \hat{G}) \sim (g, h, \hat{I}) \succ X \succ (b, l, \hat{G}) \sim (b, l, \hat{I}) \quad (3.4)$$

As to X , I assume that all proposers prefer \hat{G} receivers irrespective of L and I and that all receivers prefer \hat{I} receivers irrespective of L and I . Writing X_p for the possible values X can take for proposers given this assumption and similarly X_r for receivers we thus have:

$$X_p \in \left\{ \left[(b, h, \hat{G}) \succ (g, l, \hat{G}) \succ (b, h, \hat{I}) \succ (g, l, \hat{I}) \right], \right. \\ \left. \left[(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I}) \right] \right\} \quad (3.5)$$

$$X_r \in \left\{ \left[(b, h, \hat{I}) \succ (g, l, \hat{I}) \succ (b, h, \hat{G}) \succ (g, l, \hat{G}) \right], \right. \\ \left. \left[(g, l, \hat{I}) \succ (b, h, \hat{I}) \succ (g, l, \hat{G}) \succ (b, h, \hat{G}) \right] \right\} \quad (3.6)$$

Let a be the frequency of receivers with preferences given by Equation 3.4 and the first element of X_r in Equation 3.6 and let b be the frequency of proposers with preferences given by Equation 3.4 and the first element of X_p in Equation 3.5.

The following can be shown to hold in this case:

Proposition 12 PROPERTIES OF AN EQUILIBRIUM WITH PREFERENCES OVER PREFERENCES

Given preferences take the form as in Equations 3.4, 3.5 and 3.6 there is a continuum of equilibria that is characterised by $a + b = 1$. Each of these equilibria is stable in the absence of small random shocks and unstable in their presence. In each of these equilibria there are no matchings of (b, h) with (b, h) if $P_g > P_h$

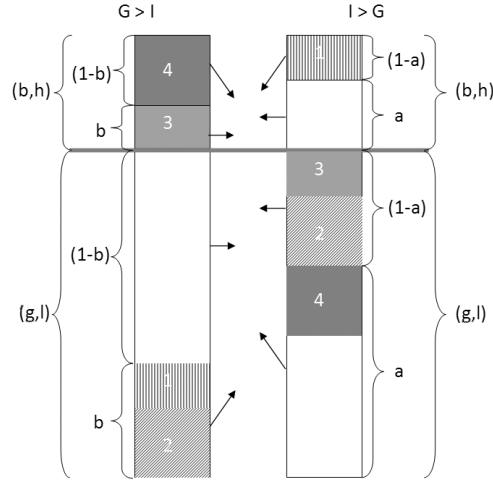


Figure 3.4: *This diagram shows an exemplary equilibrium of the Preferences over Preferences type with $a > \frac{1}{2}$ and $a + b = 1$. The numbering shows in which order one can imagine matches to occur.*

and no matchings of (g, l) with (g, l) if $P_g < P_h$ and neither of these types of matchings occur if $P_g = P_h$.

Proof: See Appendix.

This means that evolutionary efficiency is restored in the following sense: mediocre types form the maximum number of matches among themselves so that the expected reproductive difference between individuals carrying g (h) and b (l) is maximised. The reason is that even though there is no equilibrium that would be stable in the face of small random shocks, preferences will after each shock be driven to an equilibrium that ensures the maximum number of cross matches of types (g, l) and (b, h) . And again the outcome mirrors that of the benchmark scenario.

One such equilibrium is illustrated in Figure 3.4.

3.2.4 Evolution of Institutions and Co-Evolution

I assume that the evolution of institutional setups is relatively slow compared to the evolution of preferences, so that it is appropriate to compare the performance of these setups taking preferences in communities following a certain institutional setup as given by the equilibrium they approach in such setup.

By Propositions 8 and 9 the gender ratio is constant at 1 : 1 and the frequency of the better traits g and h is weakly increasing under all institutional setups and given any distribution of preferences. Differential reproductive success of two communities which adhere to different institutional setups I take to depend on how fast L and I are evolving in these communities⁶. Clearly a community with higher frequency of the favourable traits h and g will outgrow a community with a lower frequency; eventually we would thus expect to see institutions that lead to higher frequencies P_g and P_h prevail. If we take the initial distribution of traits to be equal (at least in expectation) across communities, then institutions that lead to faster evolution of L and I will eventually displace other institutions.

Communities following the Proposer Scheme will outgrow communities following a random allocation mechanism. The underlying reason is that the former makes use of more information in the matching process: (g, h) types only match with (g, h) types and (b, l) types only with (b, l) types; we therefore have a clear differential in expected reproductive success for individuals carrying h (g) versus l (b) and thus the evolution of I and L is faster. This fitness spread based argument is elaborated e.g. in Sloman and Sloman [1988].

In the case of $P_g = P_h$ the DA procedure induces the highest possible differential between expected reproductive success for g (h) and b (l) so that it weakly dominates any other allocation mechanism⁷.

This is co-evolution because institutions shape the evolution of preferences and the evolution of preferences makes for reproductive differences between institutions thus setting in motion and determining the evolution of institutions.

Within communities following the DA procedure, those that mutate to changes of the sort of ‘preferences over preferences’ will outgrow those that do not if Proposition 11 applies, i.e. if the frequencies of g and h are sufficiently different.

⁶Alternatively, one could track the total number of expected surviving children in a community of a given size under different institutional setups. For that kind of analysis, however, we need specific assumptions on both the magnitudes of $sl_x(g, h)$ versus $sl_x(g, l)$ and $sl_x(b, l)$ and the elasticity of S with respect to sl . In the longer run these effects will be dominated by the effect of differences in P_g and P_h across communities.

⁷When $P_g \neq P_h$, it can be shown that the Proposer Scheme results in a higher fitness differential than allocation mechanisms that let individuals of one gender choose in random order or grouped by either their phenotype in I or in L .

3.2.5 An Application: Why Women and Men differ in their Mating Preferences and why Women Don't Marry Down

The implications of the above analysis can be applied to modern marriage markets. The analysis is relevant if men and women today are constrained in their choice of a partner by their innate mating preferences and if these are inherited with no substantial change from our ancestors who lived before the neolithic revolution. As an important caveat of what follows I should note that the model developed above explicitly rules out any importance of culturally determined characteristics of individuals (such as status and wealth), whereas in modern human societies these seem to play a substantial role in mating markets.

Interpret I as intelligence and L as looks which I take as a proxy for underlying health. As argued above we should expect the DA procedure, being more conducive to the working of natural selection, to have prevailed over alternative schemes. Let us further assume that the frequencies of good looks g and high intelligence h are not too different so that Proposition 10 applies.

Extrapolating from the experience in historic times, one can then argue that it is more plausible to find men in the position of proposers and women in the one of receivers; and further that men clearly value beauty more than intelligence while for women it may be the other way round. In the language of the model this would translate into an equilibrium of type $\{a=1, b=0\}$.

The model does not give an account other than arbitrary initial distributions of preferences for whether it should be men who look for beauty and women for cleverness or vice versa. This initial distribution may have been tilted in favour of the observed equilibrium type because, and this is outside the model, intelligence can be argued to be more important to the success in life for men than for women⁸. From an initial distribution of preferences where women paid

⁸One of the fundamental differences between women and men, if not the most fundamental, is that only women give birth to children. While the first nine months in the life of a human being as well as the quality and abundance of nutrition in the months after its birth have been shown to be highly important to its whole later life, these in turn depend on the status of health of its mother. While intelligence helps securing food and shelter etc. for the mother, her 'underlying health' such as the strength of her immune system is also vitally important. A further reason is that, in times of high fertility and high child mortality especially, underlying

a little more attention to cleverness and rank⁹ than men and men a little more attention to looks, the forces described in Lemmata 9 and 10 transformed this distribution such that women basically uniformly value cleverness more and men value looks more¹⁰.

Co-evolution of preferences and institutions lets mating preferences be necessarily different between genders even though, first, there may be very little intrinsic difference in the (evolutionary) value of the characteristics over which preferences are defined and, second, this value is completely independent of gender.

A further question I am concerned with in this application is why, according to anecdotal evidence, highly educated women who are successful in their careers seldom marry down in the sense of marrying a man with markedly less education or a markedly worse career while this is not uncommon for successful and well educated men.

The answer this paper offers is the following: in equilibrium our ancestors matched clever men with good looking women and clever women with good looking men with the help of a proposal based institution that shaped preferences such that the clever men and good looking women wanted their match while the clever women and good looking men preferred the respective other type. This wrong preference did not matter because the concerned individuals realised that they had no better choice. In order to realise this, however, it is necessary to know the mating market well (as is assumed in the model). The modern day equivalents of the members of the forced pairs, good looking men and clever women, have maladapted preferences in the sense that they are programmed to pair up with the not preferred type only if they are sure that there is no preferred partner available in the market. And to be sure is much more

health is important to how well the woman takes the high toll of pregnancies and childbirth and thus how well she is prepared for subsequent pregnancies and the raising of her children. For men on the other hand, no such direct connection between bodily health of father and child exists. What matters is whether they can provide goods like food and shelter; and this, above a certain minimum threshold of health can be argued to depend more on intelligence than health.

⁹Above a certain minimum level of health, intelligence related social and emotional skills, the ability to think strategically etc. can be argued to be the decisive factors in the determination of social rank.

¹⁰In terms of outcome, Bjerk [2009] reaches similar conclusions based on a model of human capital investment that determine wealth, a cultural characteristic.

difficult in the vastly greater and much more varied marriage markets today. It is because of the mating market of times long gone that women today do not marry down and instead continue their search for the elusive intelligent man.

3.3 Discussion

Within the framework of a monogamous mating market, the model of this paper demonstrates that nature may delegate information processing to a culturally transmitted institutional setup. Genes and culture co-evolve towards an evolutionarily efficient solution to the complexity constraint introduced in the genetic setup of mating preferences.

The DA procedure has been shown to be a plausible institutional setup to emerge replacing a random allocation setup. At the same time it seems reasonable to say that the cognitive capacities required of the individuals in the DA procedure are higher than in simpler institutional setups such as random allocation - in particular if individuals are expected to know whom it is futile to propose to and whom one should accept immediately. We may thus find that species of higher such capacities are more likely to follow something resembling the DA procedure.

While the sophistication of the institutional setup is thus constrained by the intelligence of the species, the fact that more sophisticated institutions provide advantages will encourage the development of greater cognitive capacities. While this is obvious in the primate and human domain, the merit of this model is to show this complementarity in a very simple and more widely applicable setup.

As for human mating preferences¹¹, the model suggests that men and women may have different rankings over traits whose reproductive value is a priori independent of gender. A more speculative interpretation is that men may value looks more than intelligence and its correlates whereas for women it is the other way round. Adding uncertainty over market conditions may then prevent women from opting to marry down the social ladder while this seems to present little problems for many men.

¹¹The literature on human mating preferences is large; for an introductory survey see for instance Roberts and Little [2008].

Bibliography

- M. Andersson. *Sexual Selection*. Princeton University Press, Princeton, 1994.
- L.S. Bagwell and B.D. Bernheim. Veblen effects in a theory of conspicuous consumption. *The American Economic Review*, 86(3):349–373, 1996.
- G. Becker. An economic analysis of fertility, in Demographic and Economic Change in Developed Countries. In *Conference of the Universities, National Bureau Committee for Economic Research, Princeton University Press, Princeton*, pages 209–240, 1960.
- G.S. Becker. A Treatise on the Family. *Harvard University Press, Cambridge, MA*, 1981.
- G.S. Becker and R.J. Barro. A reformulation of the economic theory of fertility. *The Quarterly Journal of Economics*, 103(1):1–25, 1988.
- G.S. Becker and H.G. Lewis. On the Interaction between the Quantity and Quality of Children. *The Journal of Political Economy*, 81(2):279–288, 1973.
- G.S. Becker and N. Tomes. Child endowments and the quantity and quality of children. *Journal of Political Economy*, 84(4):143–162, 1976.
- G.S. Becker and N. Tomes. An equilibrium theory of the distribution of income and intergenerational mobility. *The Journal of Political Economy*, 87(6):1153–1189, 1979.
- T. Bergstrom. Primogeniture, monogamy, and reproductive success in a stratified society. unpublished, 1994.
- B.D. Bernheim, A. Shleifer, and L.H. Summers. The strategic bequest motive. *The Journal of Political Economy*, 93(6):1045–1076, 1985.

- G. Bertocchi. The law of primogeniture and the transition from landed aristocracy to industrial democracy. *Journal of Economic Growth*, 11(1):43–70, 2006.
- D. Bjerk. Beauty vs. earnings: Gender differences in earnings and priorities over spousal characteristics in a matching model. *Journal of Economic Behavior & Organization*, 69(3):248–259, 2009.
- S. Bowles. Group competition, reproductive leveling, and the evolution of human altruism. *Science*, 314(5805):1569–1572, 2006.
- S. Bowles, J.K. Choi, and A. Hopfensitz. The co-evolution of individual behaviors and social institutions. *Journal of Theoretical Biology*, 223(2):135–147, 2003.
- R. Boyd and P.J. Richerson. Group beneficial norms can spread rapidly in a structured population. *Journal of Theoretical Biology*, 215(3):287–296, 2002.
- L.L. Cavalli-Sforza and M.W. Feldman. *Cultural transmission and evolution: A quantitative approach*. Princeton University Press, Princeton, 1981.
- J.K. Choi and S. Bowles. The coevolution of parochial altruism and war. *Science*, 318(5850):636–640, 2007.
- C.Y.C. Chu. Primogeniture. *Journal of Political Economy*, 99(1):78–99, 1991.
- H.L. Cole, G.J. Mailath, and A. Postlewaite. Social norms, savings behavior, and growth. *Journal of Political Economy*, 100(6):1092–1125, 1992.
- J.W. Cole and E.R. Wolf. *The hidden frontier: ecology and ethnicity in an Alpine valley*. Academic Press, New York, 1974.
- C. Darwin. *Descent of Man, and Selection in Relation to Sex*. John Murray, London, 1871.
- J.S. Duesenberry. *Income, Saving and the Theory of Consumer Behavior*. Harvard University Press, Cambridge, MA, 1949.
- W.H. Durham. *Coevolution: Genes, culture, and human diversity*. Stanford University Press, Stanford, 1992.

- R.A. Easterlin. The conflict between aspirations and resources. *Population and Development Review*, 2(3):417–425, 1976.
- R.L. Faith. Inheritance, equal division and rent seeking. ASU Economics Working Paper No. 15/2001, 2001.
- R.L. Faith, B.L. Goff, and R.D. Tollison. Bequests, sibling rivalry, and rent seeking. *Public Choice*, 136(3):397–409, 2008.
- M.W. Feldman and L.A. Zhivotovsky. Gene-culture coevolution: Toward a general theory of vertical transmission. *Proceedings of the National Academy of Sciences of the United States of America*, 89(24):11935–11938, 1992.
- RA Fisher. *The genetical theory of natural selection*. Clarendon Press, Oxford, 1930.
- R.H. Frank. *Choosing the right pond: Human behavior and the quest for status*. Oxford University Press, New York, 1985a.
- R.H. Frank. The demand for unobservable and other nonpositional goods. *The American Economic Review*, 75(1):101–116, 1985b.
- R.H. Frank. Positional externalities cause large and preventable welfare losses. *American economic review*, 95(2):137–141, 2005.
- D. Gale and L.S. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.
- J. Gali. Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices. *Journal of Money, Credit & Banking*, 26(1):1–8, 1994.
- J.H. Gillespie. Natural selection for variances in offspring numbers: a new evolutionary principle. *American Naturalist*, 111(981):1010–1014, 1977.
- H. Gintis. A framework for the unification of the behavioral sciences. *Behavioral and Brain Sciences*, 30(01):1–16, 2007.
- A. Grafen. Sexual selection unhandicapped by the Fisher process. *Journal of Theoretical Biology*, 144(4):473–516, 1990.

- P.L.E. Grieco and N.L. Ziebarth. Unigeniture in an Uncertain World. working paper, 2010.
- guardian.co.uk. Should we have more children?
<http://www.guardian.co.uk/money/2006/may/12/personaleffects>,
 2006.
- W.T. Harbaugh. What do donations buy?: A model of philanthropy based on prestige and warm glow. *Journal of Public Economics*, 67(2):269–284, 1998.
- F. Hirsch. *Social limits to growth*. Taylor & Francis Group, 1978.
- E. Hopkins and T. Kornienko. Running to keep in the same place: consumer choice as a game of status. *American Economic Review*, 94(4):1085–1107, 2004.
- S.B. Hrdy and D.S. Judge. Darwin and the puzzle of primogeniture. *Human Nature*, 4(1):1–45, 1993.
- N.J. Ireland. On limiting the market for status signals. *Journal of Public Economics*, 54(1):91–110, 1994.
- S. J. Solnick and D. Hemenway. Is more always better?: A survey on positional concerns. *Journal of Economic Behavior and Organization*, 37(3):373–383, 1998.
- S. Karlin and P. O’Donald. Some population genetic models combining sexual selection with assortative mating. *Heredity*, 41:165–174, 1978.
- D.E. Knuth. *Stable marriage and its relation to other combinatorial problems: An introduction to the mathematical analysis of algorithms*. American Mathematical Society, 1997.
- R. Lande. Models of speciation by sexual selection on polygenic traits. *Proceedings of the National Academy of Sciences of the United States of America*, 78(6):3721, 1981.
- P. Legros and A.F. Newman. Beauty is a beast, frog is a prince: assortative matching with nontransferabilities. *Econometrica*, 75(4):1073–1102, 2007.

- O. Moav and Z. Neeman. Conspicuous Consumption, Human Capital and Poverty. working paper, 2010.
- M. Myrskylä, H.P. Kohler, and F.C. Billari. Advances in development reverse fertility declines. *Nature*, 460(7256):741–743, 2009.
- C. Nakane. *Kinship and economic organization in rural Japan*. Athlone P., 1967.
- D. Neumark and A. Postlewaite. Relative income concerns and the rise in married women’s employment. *Journal of Public Economics*, 70(1):157–183, 1998.
- A. Pomiankowski and Y. Iwasa. Evolution of multiple sexual preferences by Fisher’s runaway process of sexual selection. *Proceedings: Biological Sciences*, 253(1337):173–181, 1993.
- A. Postlewaite. The social basis of interdependent preferences. *European Economic Review*, 42(3):779–800, 1998.
- L. Rayo and G.S. Becker. Evolutionary efficiency and happiness. *Journal of Political Economy*, 115(2):302–337, 2007.
- S.C. Roberts and A.C. Little. Good genes, complementary genes and human mate preferences. *Genetica*, 134(1):31–43, 2008.
- A.E. Roth and M.A.O. Sotomayor. *Two-sided matching: A study in game-theoretic modeling and analysis*. Cambridge University Press, Cambridge, 1992.
- R.K. Sah. The effects of child mortality changes on fertility choice and parental welfare. *Journal of Political Economy*, 99(3):582–606, 1991.
- L. Samuelson. Information-based relative consumption effects. *Econometrica*, 72(1):93–118, 2004.
- M.R. Servedio and R. Lande. Population genetic models of male and mutual mate choice. *Evolution*, 60(4):674–685, 2006.
- E. Sheshinski and Y. Weiss. Inequality within and between families. *The Journal of Political Economy*, 90(1):105–127, 1982.
- S. Sloman and L. Sloman. Mate selection in the service of human evolution. *Journal of Social and Biological Systems*, 11(4):457–468, 1988.

- A. Smith. *An Inquiry into the Nature and Causes of the Wealth of Nations*. W. Strahan and T. Cadell, London, 1776.
- S.J. Solnick and D. Hemenway. Are positional concerns stronger in some domains than in others? *American Economic Review*, 95(2):147–151, 2005.
- J. Soltis, R. Boyd, and P.J. Richerson. Can group-functional behaviors evolve by cultural group selection?: An empirical test. *Current Anthropology*, 36(3):473–494, 1995.
- F. Tournemaine. Social aspirations and choice of fertility: why can status motive reduce per-capita growth? *Journal of Population Economics*, 21(1):49–66, 2008.
- Thorstein Veblen. *The Theory of the Leisure Class*. The Macmillan Company, 1899.
- D.S. Wilson and L.A. Dugatkin. Group selection and assortative interactions. *American Naturalist*, 149(2):336–351, 1997.
- A. Zahavi. Mate selection—a selection for a handicap. *Journal of theoretical Biology*, 53(1):205–214, 1975.

Appendices

Second Order Conditions for the continuous n case (Chapter 1)

We need to show that the matrix of second derivatives is negative definite. I test the leading principal minors.

Writing V_{ij} for $\frac{\partial^2 V}{\partial i \partial j}$ and making use of the first order conditions (equations (1.5) and (1.4)), note that:

$$\begin{aligned} V_{nn} &= (b_i + \beta)^2 u''(\cdot) r_{p,i} - \epsilon n_i^{-1} u'(\cdot) r_{p,i} < 0 \\ V_{bb} &= n_i^2 u''(\cdot) r_{p,i} + \alpha n_i^{1-\epsilon} [u''(\cdot) f'(\cdot) + u'(\cdot) f''(\cdot)] r_{c,i} < 0 \\ V_{nb} &= V_{bn} = (b_i + \beta) n_i u''(\cdot) r_{p,i} + (1 - \epsilon) u'(\cdot) r_{p,i} \end{aligned}$$

Note further that by the condition imposed in inequality (1.8) and the assumption of $1 - \epsilon < 1$ we know that $u(f(b_i))$ is a concave function, so $V_{bb} < 0$.

The leading principal minors must be alternating in sign, so we need $V_{nn} < 0$ and $V_{nn} V_{bb} - V_{bn}^2 > 0$. The first of these conditions is automatically satisfied; the second is satisfied iff:

$$V_{nn} \alpha n_i^{1-\epsilon} [u''(\cdot) f'(\cdot) + u'(\cdot) f''(\cdot)] r_{c,i} - (2 - \epsilon) n_i (b_i + \beta) u''(\cdot) u'(\cdot) r_{p,i}^2 - (1 - \epsilon)^2 u'(\cdot)^2 r_{p,i}^2 > 0$$

As can readily be seen, only the last summand is negative; and this inequality is satisfied 'in excess' if the following holds:

$$-(2 - \epsilon) n_i (b_i + \beta) u''(\cdot) - (1 - \epsilon)^2 u'(\cdot) > 0$$

In words, this tells us that $u(\cdot)$ must be sufficiently concave. The intuition is that a lack of curvature may result in a corner solution where all income is spent on parent consumption because then the weighting by $\alpha < 1$ renders investment into children uninteresting. It is assumed throughout that the last inequality holds.

Proof of inequality (1.9), n_i increasing in parent income

Label the first order condition with respect to n_i , which is given in equation (1.4), as Q . Then from the implicit function theorem and from noting that $Q = \frac{\partial V}{\partial n_i}$ we get:

$$\frac{\partial n_i}{\partial z_i} = -\frac{(\frac{\partial Q}{\partial z_i})}{(\frac{\partial Q}{\partial n_i})} = -\frac{\frac{\partial^2 V}{\partial n_i \partial z_i} + \frac{\partial^2 V}{\partial n_i \partial b_i} \cdot \frac{\partial b_i}{\partial z_i}}{\frac{\partial^2 V}{\partial n_i^2}} > 0$$

It is immediate that $\frac{\partial^2 V}{\partial n_i \partial z_i} > 0$ and $\frac{\partial^2 V}{\partial n_i^2} < 0$, and it has been assumed that $\frac{\partial b_i}{\partial z_i} > 0$. By using equation (1.5), we can show that

$$\frac{\partial^2 V}{\partial n_i \partial b_i} = (b_i + \beta)n_i u''(\cdot)r_{p,i} + (1 - \epsilon)u'(\cdot)r_{p,i}$$

This derivative is assumed negative which is a slightly stronger concavity assumption on $u(\cdot)$ compared to the discussion of the second order conditions for the continuous n case with two generations.

Rearranging the first equation of this proof then yields the condition on $\frac{\partial b_i}{\partial z_i}$ written in (1.9). \square

Proof of Lemma 1

For given values of n and $r_{c,i}$, the only choice variable of the agent is b . The first order condition is Equation (1.5), label it Q for this proof; the second order condition is satisfied as V_{bb} has been shown to be negative.

Using the implicit function theorem we arrive at the following relationships:

$$\begin{aligned} \frac{\partial b_i^e(n, r_c)}{\partial z_i} &= -\frac{(\frac{\partial Q}{\partial z_i})}{(\frac{\partial Q}{\partial b_i^e(n, r_c)})} = -\frac{-n_i u''(\cdot)r_{p,i}}{V_{bb}} > 0 \\ \frac{\partial b_i^e(n, r_c)}{\partial r_c} &= -\frac{(\frac{\partial Q}{\partial r_c})}{(\frac{\partial Q}{\partial b_i^e(n, r_c)})} = -\frac{\alpha n_i^{1-\epsilon} u'(\cdot) f'(\cdot)}{V_{bb}} > 0 \end{aligned}$$

We have set $V_{bn} < 0$ - see the proof of inequality (1.9) - so were n_i continuous, then $b_i^e(n, r_c)$ would vary with n_i according to $\frac{\partial b_i^e(n, r_c)}{\partial n_i} = -\frac{V_{bn}}{V_{bb}} < 0$. Now n_i is discrete and moving from n to $n+1$ will translate into

$$b_i^e(n+1, r_c) - b_i^e(n, r_c) = \int_n^{n+1} \frac{\partial b_i^e(x, r_c)}{\partial x} dx < 0$$

So $b_i^e(n, r_c)$ is decreasing in n . □

Proof of Lemma 2

The net marginal benefit of an additional n^{th} child ($NMBC$) is given by:

$$\begin{aligned} NMBC &= \text{marginal benefit} - \text{marginal cost} \\ &= \alpha n^{-\epsilon} u(f(b)) r_c - \left[\left(u(z_i - (n-1)(b + \beta)) - u(z_i - n(b + \beta)) \right) r_p \right. \\ &\quad \left. + \alpha(n-1) \left((n-1)^{-\epsilon} - n^{-\epsilon} \right) u(f(b)) r_c \right] \end{aligned}$$

Marginal benefit is the added well-being of the additional n^{th} child while marginal cost can be decomposed into the utility loss from lower parent consumption and the dilution effect on the altruistic feelings of the agent towards the “existing” $n-1$ children.

It is easily shown that $NMBC$ is a strictly decreasing function of n , so when considering marginal changes in the factors that influence $n_i^e(b, r_c)$ it suffices to check whether $n_i^e(b, r_c)$ changes to $n_i^e(b, r_c) + / - 1$.

Accept for the moment the shorthand of $n = n_i^e(b, r_c)$. Then we can see that:

$n-1$		n		$n+1$
$V(z_i n-1)$	\leq	$V(z_i n)$	\geq	$V(z_i n+1)$
$\frac{\partial V(z_i n-1)}{\partial z_i}$	$<$	$\frac{\partial V(z_i n)}{\partial z_i}$	$<$	$\frac{\partial V(z_i n+1)}{\partial z_i}$
$\frac{\partial V(z_i n-1)}{\partial r_c}$	$<$	$\frac{\partial V(z_i n)}{\partial r_c}$	$<$	$\frac{\partial V(z_i n+1)}{\partial r_c}$

The inequalities stem from the facts that, firstly, $\frac{\partial V(z_i|n)}{\partial z_i} = u'(z_i - n(b + \beta)) r_{p,i}$

is increasing in n , and secondly, that $\frac{\partial V(z_i|n)}{\partial r_c} = \alpha n^{1-\epsilon} u(f(b))$ is increasing in n . If fertility was continuous, then $\frac{\partial n_i^e}{\partial b} = -\frac{V_{nb}}{V_{nn}} < 0$. This underlying structure is left intact when the integer constraint is imposed. \square

Proof of Lemma 3

Note that for positive N_i , equilibrium utility for the parent agent given $B_i = b_i^e(r_{c,i})$ is weakly greater than \underline{u}_i ; also note that $\frac{\partial V(z_i)}{\partial b}|_{b=b_i^e(r_{c,i})} = 0$ while $\frac{\partial^2 V(z_i)}{\partial b^2} < 0$. Hence if b can take any positive value, \exists a unique $b > b_i^e(r_{c,i})$ for which $V(z_i|b=b) = \underline{u}_i$, call this $b \bar{B}_i$. In the model b is bounded above by the feasibility constraint $b_i \leq z_i - \beta$, therefore we define \bar{B}_i as done in the Lemma.

Was $B_i > \bar{B}_i$ then a deviation to the option that yields utility \underline{u}_i would be profitable and the putative equilibrium would be destroyed.

$\bar{B}_i > b_i^e(r_{c,i})$ because of the inequality mentioned at the beginning of this proof and the smooth process by which utility falls in b ; note also that $b_i^e(r_{c,i})$ is always smaller than $z_i - \beta$ because of the concavity of $u(\cdot)$.

Note that for N_i unchanged and \bar{B}_i not being equal to $z_i - \beta$ we have the following relationships:

$$\begin{aligned} \frac{\partial \bar{B}_i}{\partial r_{c,i}} &= -\frac{\left(\frac{\partial Q}{\partial r_{c,i}}\right)}{\left(\frac{\partial Q}{\partial \bar{B}_i}\right)} = -\frac{\frac{\partial V(z_i)}{\partial r_{c,i}}}{\frac{\partial V(z_i)}{\partial b}|_{b>b_i^e(r_{c,i})}} > 0 \\ \frac{\partial \bar{B}_i}{\partial z_i} &= -\frac{\left(\frac{\partial Q}{\partial z_i}\right)}{\left(\frac{\partial Q}{\partial \bar{B}_i}\right)} = -\frac{u'(z_i - N_i(B_i + \beta)) \cdot r_{p,i} - u'(z_i - n_i^e(r_{c,j}, \check{B}_j)(\check{B}_j + \beta)) \cdot r_{p,i}}{\frac{\partial V(z_i)}{\partial b}|_{b>b_i^e(r_{c,i})}} \end{aligned}$$

The sign of the latter quotient clearly is given by the sign of:

$$N_i(B_i + \beta) - n_i^e(r_{c,j}, \check{B}_j)(\check{B}_j + \beta) \quad \square$$

Proof of Proposition 3

Let $k = 1$ and note that as the agent would prefer $n_i^e(r_{c,i})$ to $n_i^e(r_{c,i})-1$ we have $Q(b_i^e(r_{c,i}, n=n_i^e(r_{c,i}) - k + 1)) < 0$. As b increases, the first summand of Q will initially grow because $b_i^e(r_{c,i}, n)$ is decreasing in n and eventually fall as

$b_i^e(r_{c,i}, n=n_i^e(r_{c,i}) - k)$ is surpassed; the second summand, however, will fall from the start. We can show that Q is increasing in $b > b_i^e(r_{c,i}, n=n_i^e(r_{c,i}) - k + 1)$: $\frac{\partial Q}{\partial b} = \frac{\partial V(n_i^e(r_{c,i})-k)}{\partial b} - \frac{\partial V(n_i^e(r_{c,i})-k+1)}{\partial b} \approx \frac{\partial V(n_i^e(r_{c,i})-k)}{\partial b} (1 - (1 + V_{bn})) > 0$. Hence \exists a unique $\tilde{b}_{i,k}$. One can see that $\tilde{b}_{i,k} > b_i^e(r_{c,i})$, lower values need not be considered as from Lemma 4 we know that $B_i \geq b_i^e(r_{c,i})$.

Applying the above logic recursively with higher and higher k it becomes clear that $\tilde{b}_{i,k}$ is increasing in k ; the driving force behind this result is the fact that, as mentioned, $b_i^e(r_{c,i}, n)$ is decreasing in n .

The fertility choice of agents given $\min\{\bar{B}_i, \tilde{b}_{i,k+1}\} \geq B_i > \tilde{b}_{i,k}$ follows from equilibrium fertility being efficient given the equilibrium parental investment level. \square

Proof of Lemma 9

Note first that receivers of a type $\in Q = \{(g, l), (b, h)\}$ will accept a proposer of the preferred type $\in Q$ because they rightly expect never to receive a proposal from a type (g, h) proposer. Note second that they will reject a proposer of the less preferred type $\in Q$ as long as there are unmatched proposers of the preferred type $\in Q$ who prefer one's own phenotype to the complement phenotype in Q ; the reason being that accepting means settling with the less preferred type in Q for certain while rejecting means getting matched with the preferred type with a positive probability and with the less preferred type with the complement probability.

Five cases must now be considered and the relative performance of the carriers of the two possible preferences assessed:

First, $a > \frac{1}{2}$ and $b < \frac{1}{2}$. Within this case two subcases can be distinguished, $1 - b \geq a$ and $1 - b < a$; the reasoning is as follows: in the first case there are more (b, h) proposers seeking a partner of type (g, l) than there are (g, l) receivers wanting a (b, h) partner, thus all these receivers will be matched with a (b, h) proposer. Likewise there are more (g, l) proposers wanting a (g, l) receiver as a partner than there are (g, l) receivers wanting a (g, l) partner so that all receivers of this type are matched with a (g, l) proposer. Note that this allocates all (g, l) receivers so that all the remaining proposers are matched to (b, h) receivers. In the second case there are more (b, h) receivers seeking a (b, h)

partner than there are (b, h) proposers seeking a (b, h) partner, so all these proposers are matched with (b, h) receiver. And likewise there are more (g, l) receivers preferring (b, h) partners than there are (b, h) proposers wanting a (g, l) , so that all these proposers are matched with (g, l) receivers. This allocates all (b, h) proposers so that the remaining receivers are matched with (g, l) proposers. Of the receivers acting according to $X = [(b, h) \succ (g, l)]$ the (g, l) types will therefore be matched rightly with probability 1 or $\frac{1-b}{a}$, depending on whether $1-b > a$ or not, while the (b, h) types will be matched wrongly with probability $\frac{1-a}{a}$ and $\frac{b}{a}$ in the respective cases. The proportion of right matches is thus either $\frac{1}{2}(1 + \frac{a-(1-a)}{a}) > \frac{1}{2}$ or $\frac{1}{2}(\frac{1-b}{a} + \frac{a-b}{a}) > \frac{1}{2}$. Of the receivers acting according to $X = [(b, h) \prec (g, l)]$ on the other hand, all of the (g, l) type are matched wrongly so that the proportion of right matches is $\leq \frac{1}{2}$. Noting that no differences in the success of life of the wrong couples (g, l) with (g, l) and (b, h) with (b, h) exist, it is the proportion of right or optimal matches that counts. We can thus conclude that the evolutionary fitness of receivers with $X = [(b, h) \succ (g, l)]$ is higher so that a increases.

Second, $a < \frac{1}{2}$ and $b < \frac{1}{2}$. Following a similar reasoning to the first case, of the receivers acting according to $X = [(b, h) \succ (g, l)]$ all the (g, l) types will be matched rightly, so that the proportion of right matches is $\geq \frac{1}{2}$. Of the receivers acting according to $X = [(b, h) \prec (g, l)]$ the (g, l) types will be matched wrongly with probability or $\frac{1-b}{1-a}$, while the (b, h) types will in the corresponding cases be matched rightly with probability $\frac{a}{1-a}$ and $\frac{b}{1-a}$. The proportion of right matches is therefore either $\frac{1}{2}(0 + \frac{a}{1-a}) < \frac{1}{2}$ or $\frac{1}{2}(\frac{(1-a)-(1-b)}{1-a} + \frac{b}{1-a}) < \frac{1}{2}$. So again carriers of $X = [(b, h) \succ (g, l)]$ fare better and a increases.

Third, $a > \frac{1}{2}$ and $b > \frac{1}{2}$. Following a similar reasoning to the first case, of the receivers acting according to $X = [(b, h) \succ (g, l)]$ the (g, l) types will be matched rightly with probability $\frac{1-b}{a}$, while the (b, h) types will be matched wrongly with probability 1 or $\frac{b}{a}$. The proportion of right matches is thus at most $\frac{1}{2}(\frac{1-b}{a} + \frac{a-b}{a}) < \frac{1}{2}$. Of the receivers acting according to $X = [(b, h) \prec (g, l)]$ all the the (h, b) types are matched rightly, so the proportion of right matches is $\geq \frac{1}{2}$. Carriers of $X = [(b, h) \prec (g, l)]$ thus have higher fitness and therefore a decreases.

Fourth, $a < \frac{1}{2}$ and $b > \frac{1}{2}$. Following a similar reasoning to the first case, of the receivers acting according to $X = [(b, h) \succ (g, l)]$ all the (b, h) types will be matched wrongly so the proportion of right matches is $\leq \frac{1}{2}$. Of the receivers

acting according to $X = [(b, h) \prec (g, l)]$ the (g, l) types will be matched rightly with probability $\frac{(1-a)-(1-b)}{1-a}$ or $\frac{(1-a)-a}{1-a}$ and the (b, h) types with probability 1 or $\frac{b}{1-a}$. The proportion of right matches is thus either $\frac{1}{2}(1 + \frac{(1-a)-(1-b)}{1-a}) > \frac{1}{2}$ or $\frac{1}{2}(\frac{b}{1-a} + \frac{(1-a)-a}{1-a}) > \frac{1}{2}$. So again carriers of $X = [(b, h) \prec (g, l)]$ fare better and a decreases.

And fifth, $a = \frac{1}{2}$ and/or $b = \frac{1}{2}$. For these cases it is straightforward to show that there is no fitness differential for either gender so that the frequencies do not change. \square

Note that the fitness condition used in the proof subsumes the success in life and mating success considerations of the above section on the ranked chooser scheme; the difference in analysis arises from the fact that we have moved from an exogenous to an endogenous ranking regime.

Proof of Proposition 11

The stable equilibria in Proposition 10 require that $a \rightarrow 1$ and $b \rightarrow 0$ if $a > \frac{1}{2} > b$ and that $a \rightarrow 0$ and $b \rightarrow 1$ if $b > \frac{1}{2} > a$; we need to show that these dynamics do not hold under the conditions mentioned above.

Imagine first the case of $a > \frac{1}{2} > b$. Within this case, two subcases need to be distinguished: $\frac{P_g}{P_h} > 1$ and $\frac{P_g}{P_h} < 1$. In the first of these subcases, when $\frac{P_g}{P_h}$ is sufficiently high, then the payoffs to proposers carrying the genotype that implies $X = [(b, h) \succ (g, l)]$, applying the same reasoning as in the proof of Lemma 9, can be written as $0x + \frac{(1-b)x}{b(1-x)}(1-x) = \frac{1-b}{b}x$ (where x is short for $\frac{P_h(1-P_g)}{P_h(1-P_g) + (1-P_h)P_g}$ which is the proportion of type (b, h) individuals in the pool of type (b, h) and (g, l) individuals) whereas the payoffs to proposers carrying $X = [(b, h) \prec (g, l)]$ are $1x + 0(1-x) = x < \frac{1-b}{b}x$. Therefore we have b increasing and not going to 0 as required for the equilibrium. We can define a $r_{upper}^1 > 1$ such that if $\frac{P_g}{P_h} > r_{upper}^1$ this holds.

In the second of these subcases, when $\frac{P_g}{P_h}$ is sufficiently low, we can write the payoff to receivers carrying $X = [(b, h) \succ (g, l)]$ as simply $(1-x)$ whereas the payoff to receivers carrying $X = [(b, h) \prec (g, l)]$ is $\frac{a(1-x)}{(1-a)x}x + 0(1-x) = \frac{a}{1-a}(1-x) > (1-x)$; hence a decreases and does not approach 1 as required for the equilibrium. We can define a $r_{lower}^1 \in (0, 1)$ such that if $\frac{P_g}{P_h} < r_{lower}^1$ this

holds.

Imagine second the case of $b > \frac{1}{2} > a$. Again two subcases can be distinguished within this case: $\frac{P_g}{P_h} > 1$ and $\frac{P_g}{P_h} < 1$. In the first of these subcases, when $\frac{P_g}{P_h}$ is sufficiently high, then the payoffs to receivers carrying $X = [(b, h) \succ (g, l)]$ is $\frac{1-a}{a}x$ and the payoffs to receivers carrying $X = [(b, h) \prec (g, l)]$ as simply $x < \frac{1-a}{a}x$ and hence a increases and does not approach 0 as required for the equilibrium. We can define a $r_{upper}^1 > 2$ such that if $\frac{P_g}{P_h} > r_{upper}^1$ this holds.

In the second of these subcases, when $\frac{P_g}{P_h}$ is sufficiently low, we can write the payoff to proposers carrying $X = [(b, h) \succ (g, l)]$ is simply $1 - x$ whereas the payoff to proposers carrying $X = [(b, h) \prec (g, l)]$ is $\frac{b}{1-b}(1 - x) > (1 - x)$ and so b decreases and does not approach 1 as required for the equilibrium. We can define a $r_{lower}^2 \in (0, 1)$ such that if $\frac{P_g}{P_h} < r_{lower}^2$ this holds.

We can now define $r_{lower} = \min\{r_{lower}^1, r_{lower}^2\}$ and $r_{upper} = \max\{r_{upper}^1, r_{upper}^2\}$.

We have shown the dynamics required for the stable equilibria in Proposition 10 do not hold if $\frac{P_g}{P_h} > r_{upper}$ or $\frac{P_g}{P_h} < r_{lower}$. \square

Proof of Proposition 12

We first have to show that equilibria take the form of $a + b = 1$. In a second step we show the claimed implications of these equilibria.

Applying the reasoning about how matches occur that was introduced in the proof of Lemma 9 and that is exemplified for the preferences underlying this proposition in 3.4 we can write the payoffs to receivers carrying the genotype that implies $X = [(b, h) \succ (g, l)]$ as $x^{\frac{a+1-b}{a}}$ (where x is short for $\frac{P_h(1-P_g)}{P_h(1-P_g)+(1-P_h)P_g}$ which is the proportion of type (b, h) individuals in the pool of type (b, h) and (g, l) individuals) and the payoffs to receivers carrying $X = [(b, h) \prec (g, l)]$ as $x^{\frac{1-a+b}{1-a}}$. We have that a increases iff $x^{\frac{a+1-b}{a}} > x^{\frac{1-a+b}{1-a}}$ and this is the case if $1 - (a + b) > 0$; likewise a decreases if $1 - (a + b) < 0$ and does not change if $1 - (a + b) = 0$. From this follows that for a given b , a will adjust such that $a + b = 1$ holds.

Similarly we can derive the payoffs to proposers carrying the genotype that implies $X = [(b, h) \succ (g, l)]$ as $x^{\frac{b+1-a}{b}}$ and the payoffs to proposers carrying $X = [(b, h) \prec (g, l)]$ as $x^{\frac{1-b+a}{1-b}}$. We have that b increases iff $x^{\frac{b+1-a}{b}} > x^{\frac{1-b+a}{1-b}}$ and this is the case if $1 - (a + b) > 0$; likewise b decreases if $1 - (a + b) < 0$ and does

not change if $1 - (a + b) = 0$. From this follows that for a given a , b will adjust such that $a + b = 1$ holds.

Taken together this implies that we have a continuum of equilibria characterised by $a + b = 1$. They are not stable because upon any shock to any one of a and b not only does the variable that has been shocked, say a to \hat{a} , respond by moving back towards an equilibrium value but also the one that has not been shocked. This latter one (b in the example) will therefore move away from its ante-shock equilibrium value towards the value that equilibrates the system given the new value of the shocked variable (\hat{a} in the example).

As for the mating allocations, we need to show that in the four general cases $P_g > P_h$ with $a > b$, $P_g \leq P_h$ with $a > b$, $P_g > P_h$ with $a \leq b$ and $P_g \leq P_h$ with $a \leq b$ matings of individuals that are both (b, h) or both (g, l) do only occur if this type is in surplus compared to the complement type in $Q = \{(b, h), (g, l)\}$. The reasoning in all the four cases is the same, therefore I only present it for the first case ($P_g > P_h$ with $a > b$) which is also depicted in Figure 3.4. Receivers of type (b, h) carrying the genotype that implies $X = [(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I})]$ will find that for their most preferred partners among the proposers (apart from the unattainable types (g, h, \hat{I}) and (g, h, \hat{G})) they are themselves the first choice (again qualified for types (g, h, \hat{I}) and (g, h, \hat{G})) and therefore these individuals match. Note that all these receivers will be matched but not all these proposers. These unmatched proposers of type (g, l) (carrying $X = [(b, h, \hat{G}) \succ (g, l, \hat{G}) \succ (b, h, \hat{I}) \succ (g, l, \hat{I})]$) will then, their most preferred partners having been cleared, propose to their second most preferred partners (receivers of type (g, l) carrying $X = [(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I})]$) for whom in turn they are the best possible match so that the proposals are accepted. Now, all these proposers are matched. With their first choice gone, receivers of type (g, l) carrying $X = [(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I})]$ will now accept proposals from their second choice proposers, type (b, h) with $X = [(b, h, \hat{G}) \succ (g, l, \hat{G}) \succ (b, h, \hat{I}) \succ (g, l, \hat{I})]$. For these proposers the first choice is also already gone so these individuals are happy to match. Since in equilibrium $(1 - a)(1 - x) = bx + [b(1 - x) - (1 - a)x]$ all receivers of type (g, l) with $X = [(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I})]$ are now matched. For proposers of type (b, h) with $X = [(g, l, \hat{G}) \succ (b, h, \hat{G}) \succ (g, l, \hat{I}) \succ (b, h, \hat{I})]$ both first and second choice partners are now fully matched and so they will propose to their

third choice, receivers of type (g, l) with $X = \left[(b, h, \hat{I}) \succ (g, l, \hat{I}) \succ (b, h, \hat{G}) \succ (g, l, \hat{G}) \right]$, for whom also the first and second choice partners are completely matched and for whom these proposers are the third choice; so these proposals are accepted. The remaining receivers are, by want of alternative, matched to type (g, l) proposers. We have shown that in equilibrium individuals of the less frequent of the two types (g, l) and (b, h) are matched with their complement in Q . \square